Effects of Heat source on Dusty Viscous Fluid over a Moving Infinite Vertical Plate with Radiation

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Abstract

Aim of this paper is to investigate the effects of heat source on MHD free convection flow of viscous fluid over an impulsively started infinite vertical plate with radiation. Heat source and radiation effects are taken into account and the dimensionless governing equation is solved using the finite difference technique. The numerical results are presented graphically for different values of the parameters entering into the problem on the velocity profiles of fluid and particles of dust, temperature and concentration profile and skin friction.

1. Introduction

Magneto convection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion, liquid-metal cooling of nuclear reactors, and electromagnetic casting of metals. In the field of power generation, MHD is receiving considerable attention due to the possibilities. It offers for much higher thermal efficiencies in power of plants. MHD finds applications in electromagnetic pumps, controlled fusion research, crystal growing, plasma jets, chemical synthesis, etc.

Radiative convective flows are encountered in countless industrial and environment process e.g. heating and cooling chambers, fossil fuel combustion energy processes , evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat transfer plays an important role in manufacturing industries for the design of reliable equipments, nuclear power plants , gas turbines and various propulsion device for aircraft, missiles, satellite and space vehicles are examples of such engineering applications.

England and emery [3] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar [7] have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate were studied by Hossain and Takhar [4] in all above studies, the stationary vertical plate is considered. Rapits and Perdikis [6] have studied the effects of thermal radiation and free convection flow past a moving infinite vertical plate, radiation effects on moving infinite vertical plate. Radiation effects on moving infinite vertical plate with variable temperature were studied by Muthucumaraswamy and Ganesan [5]. The governing equations were solved by the Laplace transform technique. Chandrakala and Antony [2] studied the effects of thermal radiation on the flow past a semi-infinite vertical isothermal plate with uniform heat flux in the presence of transversely applied magnetic field. Chandrakala [8] has studied

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thermal radiation effects on moving infinite vertical plate with uniform heat flux. Recently, Singh et. al [11] have discussed on Effect of thermal radiation on dusty viscous fluid through porous medium over a moving infinite vertical plate with uniform heat flux.

Our aim of this study is to investigate the effect of heat source on unsteady natural convection flow of dusty viscous fluid over an impulsively started infinite vertical Plate with radiation in the presence of magnetic field has not received much attention from contemporary researchers. The governing equations are solved by the finite difference technique. The velocity of fluid of dust particle, temperature, concentration profile and skin friction profiles for different parameters entering into the problem are analyzed graphically.

2.0 Nomenclature

A	:	Constant
B	:	Dusty Particle parameter
B1	:	Dusty fluids parameter
B0	:	The magnetic induction
C	:	Concentration of the fluid near the plate
C_w	:	Concentration of the plate
C_∞	:	Concentration of the fluid far away from the plate
Cp	:	Specific heat at constant pressure
D	:	The chemical molecular diffusivity
g	:	Acceleration due to gravity
Gr	:	Thermal Grashoff number
Gm	:	Modified thermal Grashoff number
k	:	Thermal conductivity of the fluid
K	:	The Stoke's resistance coefficient
Pr	:	Prandtl number
qr	:	Radiative heat flux in the y - direction
m1	:	The mass of dust particles
N	:	Radiation parameter
N0	:	The number density of the dust particles (constant)
S	:	Heat source parameter
Sc	:	Schmidt number
T	:	Temperature of the fluid near the plate
T_w	:	Temperature of the plate
T_∞	:	Temperature of the fluid far away from the plate
t	:	Time
u	:	Velocity of the fluid in the x-direction
v	:	Velocity of the dust particle in the x- direction
u0	:	Velocity of the plate
U	:	Dimensionless velocity
y	:	Coordinate axis normal to the p
y'	:	Dimensionless coordinate axis normal to the plate
k^*	:	Mean absorption coefficient

3.0 Greek symbols

α	:	Thermal diffusivity
β	:	Volumetric coefficient of thermal expansion
β'	:	Volumetric coefficient of concentration expansion
μ	:	Coefficient of viscosity
υ	:	Kinematic viscosity
ρ	:	Density
σ	:	Stefan-Boltzmann constant
τ	:	Dimensionless skin-friction
θ	:	Dimensionless temperature

4.0 Mathematical Formulation

Here the flow of an incompressible dusty viscous radiating fluid over an impulsively started infinite vertical plate with uniform heat flux in the presence of magnetic field and heat source is considered. A transverse constant magnetic field is applied *i.e.* in the direction of *y* - axis. The *x* - axis is taken along the plate in the vertical direction and the *y*-axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature in a stationary condition. At time $t > 0$, the plate is given an impulsive motion in the vertical direction against the gravitational field with constant velocity u_0. At the same time, the heat is supplied from the plate to the fluid at uniform rate. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering porous medium. Then by usual Boussinesq's approximation, the unsteady magneto hydrodynamic flow is governed by the following equation.

$$\frac{\partial u}{\partial t} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \upsilon\frac{\partial^2 u}{\partial y'^2} + \frac{KN_0}{\rho}(v - u) - \frac{\sigma B^2{}_0}{\rho}u \qquad \text{... (1)}$$

$$m_1 \frac{\partial V}{\partial t'} = K(u - v) \quad \text{... (2)}$$

$$\rho C_\rho \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + S'(T - T_\infty) \quad \text{... (3)}$$

$$\frac{\partial C}{\partial t'} = D \frac{\partial^2 C}{\partial y'^2} \quad \text{... (4)}$$

Where the rosseland approximation (Brewster [1]) is used, which leads to

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y'} \quad \text{... (5)}$$

The initial and boundary conditions are as follows

$$\left.\begin{array}{lllll} t' \le 0 : u = 0 = v, & T = T_\infty, & C = C_\infty & \text{for all} & y \\ \dot{t} > 0 : u = u_0 = v, & \dfrac{\partial T}{\partial y} = -\dfrac{q}{k}, & \dfrac{\partial C}{\partial y} = -\dfrac{j''}{D} & \text{at} & y = 0 \\ u = 0, & T \to T_\infty, & C \to C_\infty & \text{as} & y \to \infty \end{array}\right\} \quad \text{... (6)}$$

Where $A = \frac{u_0{}^2}{\upsilon}$

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expending T^4 in a Taylor series about T_∞ and neglecting higher – order terms, thus

$$T^4 \cong 4T^3{}_\infty T - 3T^4{}_\infty \quad \text{... (7)}$$

By using equation (5) and (7), equation (3) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y'^2} + S'(T - T_\infty) \ldots (8)$$

On introducing the following non-dimensional quantities

$$U = \frac{u}{u_0}, \quad V = \frac{v'}{u_0}, \quad t = \frac{t' u_0{}^2}{\upsilon}, \quad y = \frac{y' u_0}{\upsilon}$$

$$\Pr = \frac{\mu C_P}{k}, \quad N = \frac{k^* k}{4\sigma T_\infty^3}, \quad M = \frac{\sigma \upsilon B_0^2}{\rho u_0^2}, \quad Sc = \frac{\upsilon}{D}, \quad S = \frac{S' \upsilon}{u_0^2 \rho C_p}$$

$$B_1 = \frac{\upsilon K N_0}{\rho u_0^2}, \quad B = \frac{m_1 u_0^2}{VK} \quad \theta = \frac{(T' - T'_\infty)}{\left(\dfrac{q\upsilon}{k u_0}\right)}$$

$$\phi = \frac{(C' - C'_\infty)}{\left(\dfrac{j''\upsilon}{D u_0}\right)}, \quad Gr = \frac{\upsilon g \beta \left(\dfrac{q\upsilon}{k u_0}\right)}{u_0^3}, \quad Gm = \frac{\upsilon g \beta^* \left(\dfrac{j''\upsilon}{D u_0}\right)}{u_0^3} \quad \text{... (9)}$$

In Eqs. (1) to (8) leads to

$$\frac{\partial U}{\partial t} = Gr\theta + +Gm\phi + \frac{\partial^2 U}{\partial y^2} + B_1(V - U) - MU \quad \text{… (10)}$$

$$B\frac{\partial V}{\partial t} = (U - V) \quad \text{… (11)}$$

$$\frac{\partial \theta}{\partial t} = \frac{(3N+4)}{3N\,\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} + S\theta \quad \text{… (12)}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \quad \text{… (13)}$$

The initial and boundary conditions in non-dimensionless form are

$$\left.\begin{array}{llllll} t \le 0 : U = 0 = V, & \theta = 0, & \phi = 0 & \text{for all} & y \\ t > 0 : U = 1 = V, & \dfrac{\partial \theta}{\partial y} = -1, & \dfrac{\partial \phi}{\partial y} = -1 & \text{at} & y = 0 \\ U = 0, & \theta \to 0, & \phi \to 0 & \text{as} & y \to \infty \end{array}\right\} \quad \text{… (14)}$$

5.0 Solution of the problem

The governing Equations (10) to (13) are to be solved under the initial and boundary conditions of equation (14). The finite difference method is applied to solve these equations.

The equivalent finite difference scheme of equations (10) to (13) are given by

$$\left[\frac{U_{i,j+1} - U_{i,j}}{\Delta t}\right] = Gr\theta_{i,j} + Gm\phi_{i,j} + \left[\frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{(\Delta y)^2}\right]$$

$$+ B_1\left(V_{i,j} - U_{i,j}\right) - MU_{i,j} \quad \text{...(15)}$$

$$\left[\frac{V_{i,j+1} - V_{i,j}}{\Delta t}\right] = \frac{1}{B}\left(U_{i,j} - V_{i,j}\right) \quad \text{... (16)}$$

$$\left[\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t}\right] = \frac{(3N+4)}{3N\,\mathrm{Pr}}\left[\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2}\right] + S\theta_{i,j} \quad \text{... (17)}$$

$$\left[\frac{\phi_{i,j+1}-\phi_{i,j}}{\Delta t}\right]=\frac{1}{Sc}\left[\frac{\phi_{i+1,j}-2\phi_{i,j}+\phi_{i-1,j}}{(\Delta y)^2}\right] \quad \text{... (18)}$$

Here, index *i* refers to *y* and *j* to time. The mesh system is divided by taking, $\Delta y = 0.1$.

From the boundary conditions in Equation (14), we have the following equivalent.

$$\left.\begin{array}{l} U(0,0)=0=V(0,0),\ \ \theta(0,0)=0,\ \ \phi(0,0)=0 \\ U(i,0)=0=V(0,0),\ \ \theta(i,0)=0,\ \ \phi(i,0)=0,\ \text{for all } i \text{ except } i=0 \end{array}\right\} \quad \text{... (19)}$$

The boundaries conditions from equation (14) are expressed in finite difference form are as follows:

$$\left.\begin{array}{l} u(0,j)=1,\ \ \left(\frac{\partial\theta}{\partial y}\right)_{(0,j)}=-1,\ \ \left(\frac{\partial\phi}{\partial y}\right)_{(0,j)}=-1 \quad \textit{for all } j \\ u(1,j)=0,\ \ \theta(1,j)=0,\ \ \phi(1,j)=0 \quad \textit{for all } j \end{array}\right\} \quad \text{... (20)}$$

Here, infinity is taken as $y = 6$. First, the velocity of dusty fluid at the end of time step namely $U(i, j+1),\ i=1$ to 10 is computed from equation (15), the velocity of dust particle at the end of time step namely $V(i, j+1),\ i=1$ to 10 is computed from equation (16) and temperature $\theta(i, j+1), i = 1$ to 10 from equation (17) and concentration $\phi(i, j+1), i = 1$ to 10 from equation (18). The procedure is repeated until t = 1 (i.e., j = 800). During computation, Δt was chosen to be 0.00125. These computations are carried out for different values of parameters *Gr*, *Gm*, Pr, *Sc*, *M*, *N*, *B* (dust particle parameter), B_1 (dusty fluid parameter) and *t* (time). To judge the accuracy of the convergence of the finite difference scheme, the same program was run with smaller values of Δt , i.e., Δt = 0.0009, 0.001 and no significant change was observed. Hence, we conclude that the finite difference scheme is stable and convergent.

6.0 Results and Discussion

Numerical calculations have been carried out for dimensionless velocity of dusty fluid, temperature and concentration profiles for different values of parameters and are displayed in Figures-(1) to (14).

Figures-(1) to (11) represent the velocity profiles of dusty fluid for different parameters. Figure-(1) shows the variation of velocity *U* with magnetic parameter *M*. It is observed that the velocity decreases as *M* increases. From Figure-(2), it is observed that the velocity of dusty fluid increases as the Grashoff number *Gr* increase. The variation of *U* with modified Grashoff number *Gm* is shown in Figure-(3). It is noticed that increase in *Gm* leads to increase in velocity of dusty fluid. From Figure-(4) shows the variation of velocity *U* with Prandtl number Pr. It is observed that the velocity of dusty fluid decreases as Pr increases. The velocity profile of dusty fluid for Schmidt number *Sc* is shown in Figure-(5). It is clear that velocity of dusty fluid *U* decreases with increasing in *Sc*. In figure-(6), the velocity profile of dusty fluid decreases due to increasing thermal radiation parameter *N*. From Figure-(7) shows the variation of velocity profile of dusty fluid *U* with dust particle parameter *B*. It is observed that the velocity of dusty fluid decreases as *B* increases. The velocity profile of dusty fluid for B_1 (dusty fluid parameter) is shown in Figure-(8). It is clear that velocity of dusty fluid *U* decreases with increasing in B_1. The velocity profile for time variable t is shown in Figure-(9). It is clear that an increase in t leads to an increase in U. In figure-(10), the velocity profile of dusty fluid increases due to increasing heat source parameter S. From Figure-(11), it is observed that increase in Prandtl number Pr causes decrease in temperature profile of dusty fluid. Figure-(12) shows that an increase in thermal radiation parameter N causes a decrease in temperature profile of dusty fluid. In figure-(13), the temperature profile of dusty fluid increases due to increasing heat source parameter S. From Figure-(14), it is noticed that an increase in Schmidt number Sc leads to decrease in concentration profile of dusty fluid.

Figure-(15) shows the skin friction. Knowing the velocity field, the skin friction is evaluated in non-dimensional form using, $\tau = \left[-\frac{\partial u}{\partial y} \right]_{y=0}$ The numerical values of τ are calculated by applying Newton's interpolation formula for 11 points and are presented. From figure-(14), it is observed that an increase in Grashoff number Gr, Modified Grashoff number Gm, porosity parameter K_0 and thermal radiation parameter N causes decrease in skin friction, and an increase in magnetic parameter M leads an increase in skin friction.

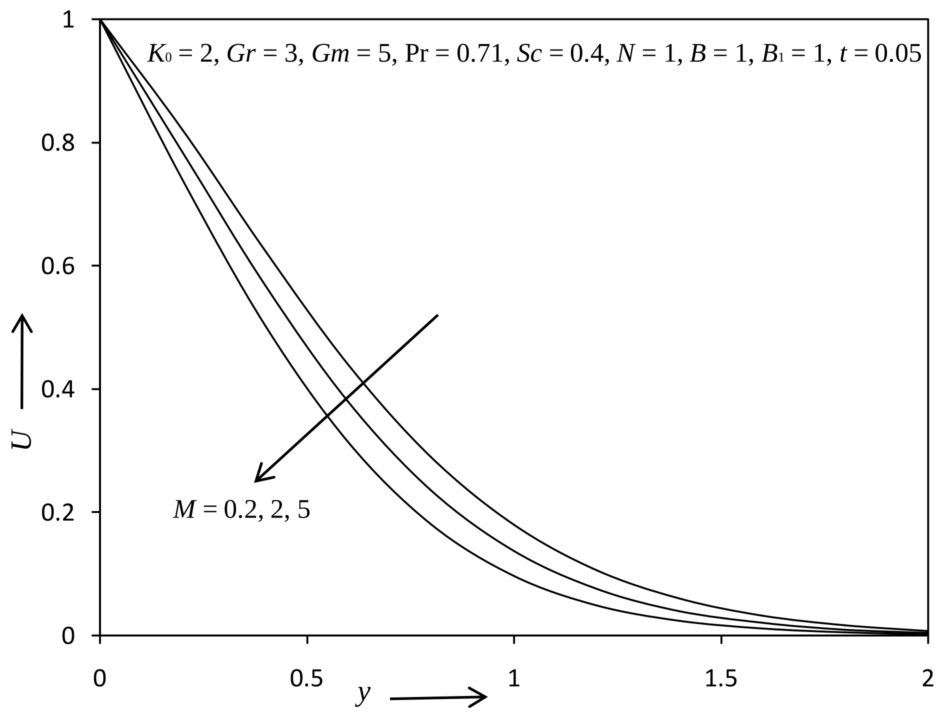

Fig: 1. The Velocity profile of dusty fluid for different value of M

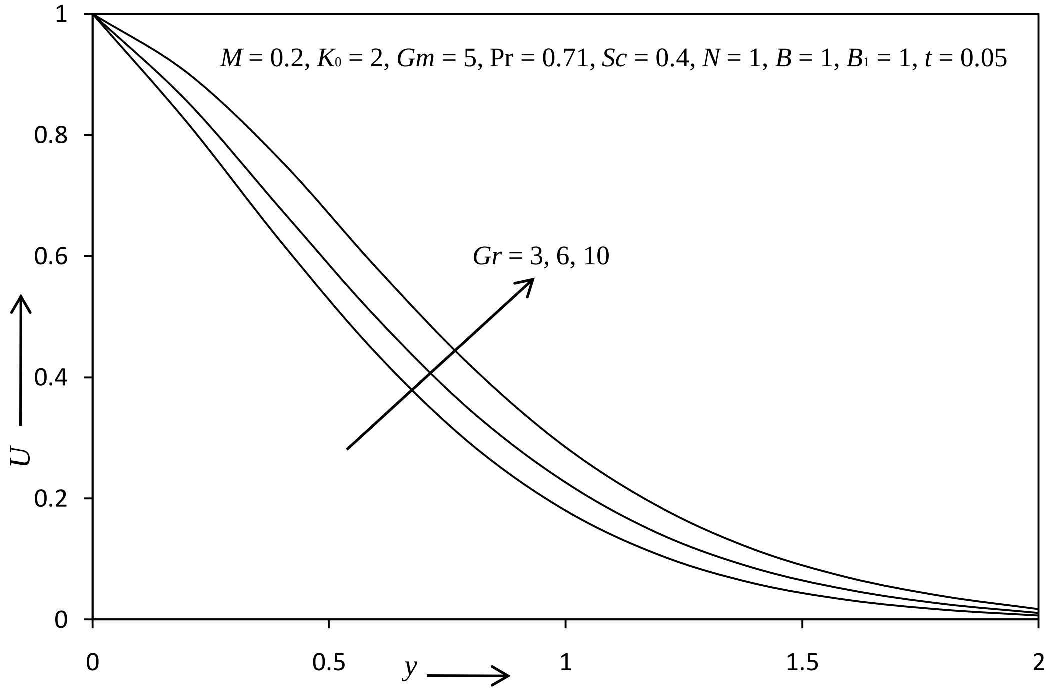

Fig: 2. The Velocity profile of dusty fluid for different value of *Gr*.

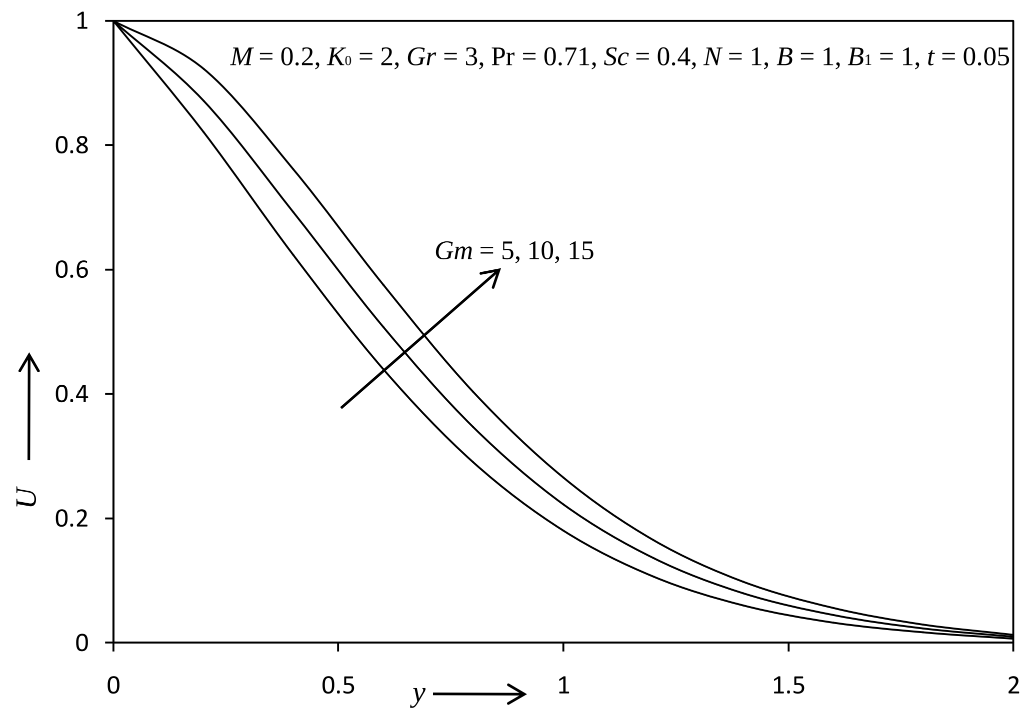

Fig: 3. The Velocity profile of dusty fluid for different value of *Gm*.

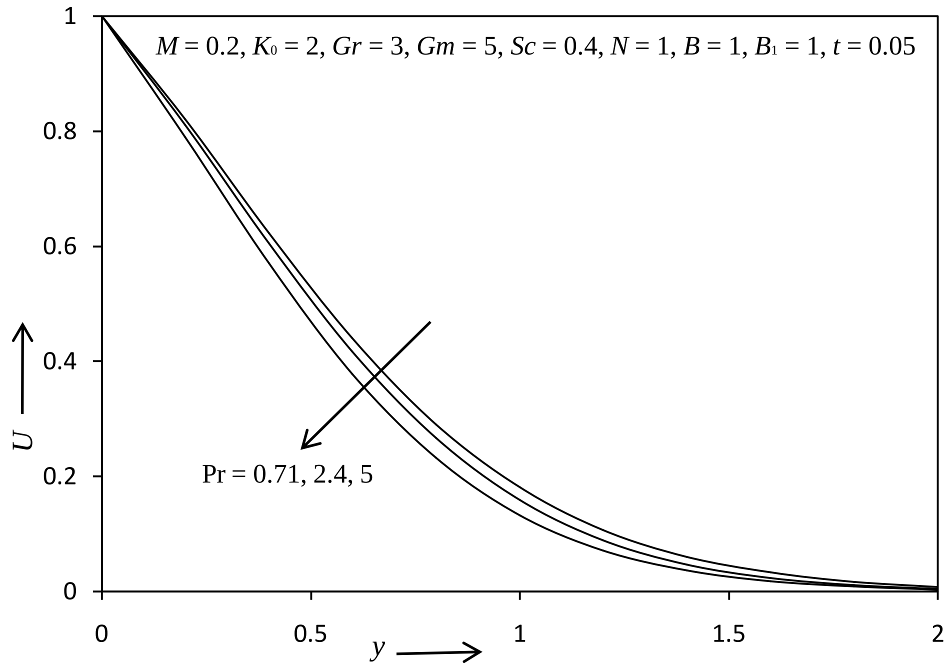

Fig: 4. The Velocity profile of dusty fluid for different value of Pr

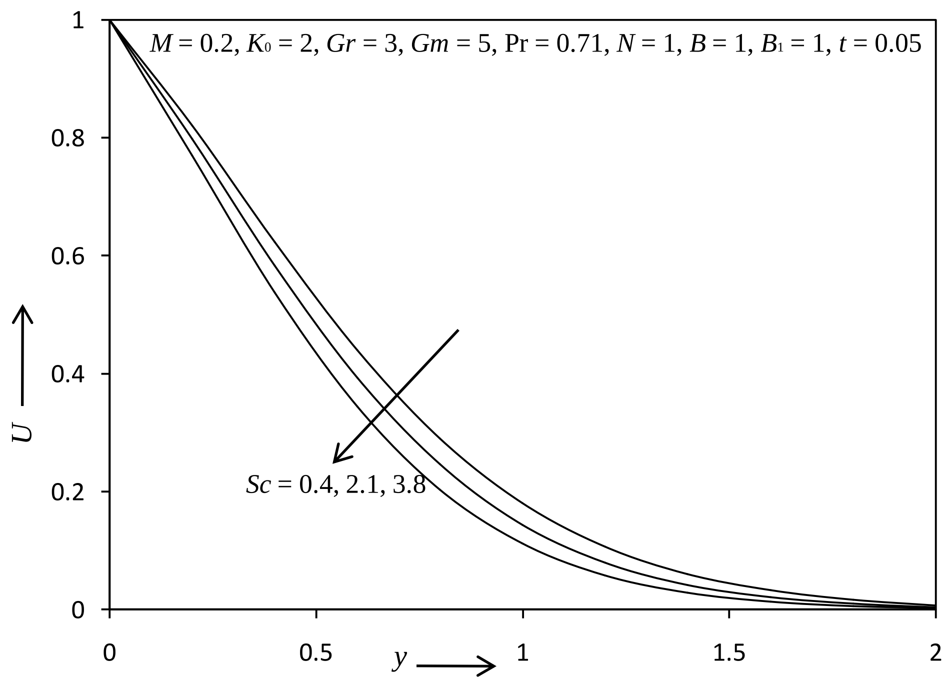

Fig: 5. The Velocity profile of dusty fluid for different value of *Sc*

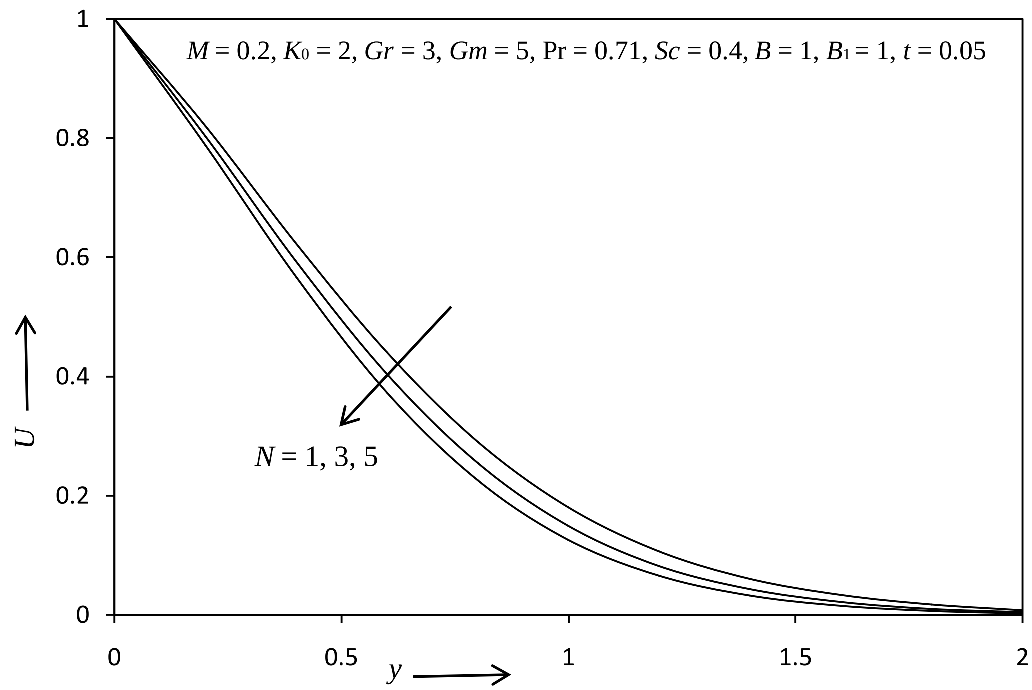

Fig: 6. The Velocity profile of dusty fluid for different value of N

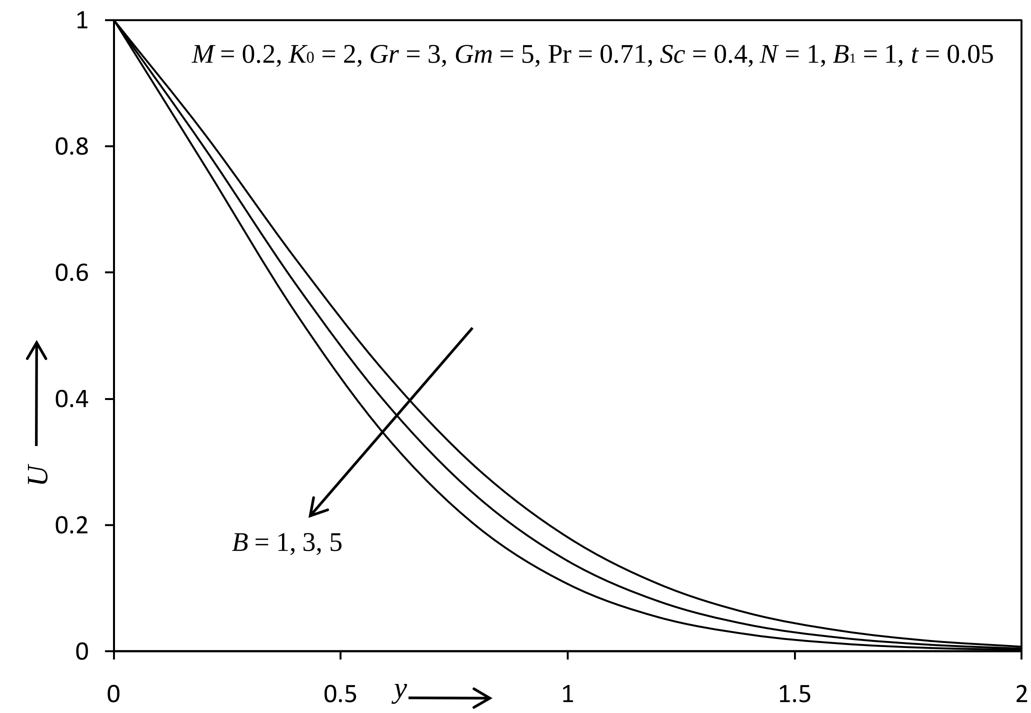

Fig: 7. The Velocity profile of dusty fluid for different value of B

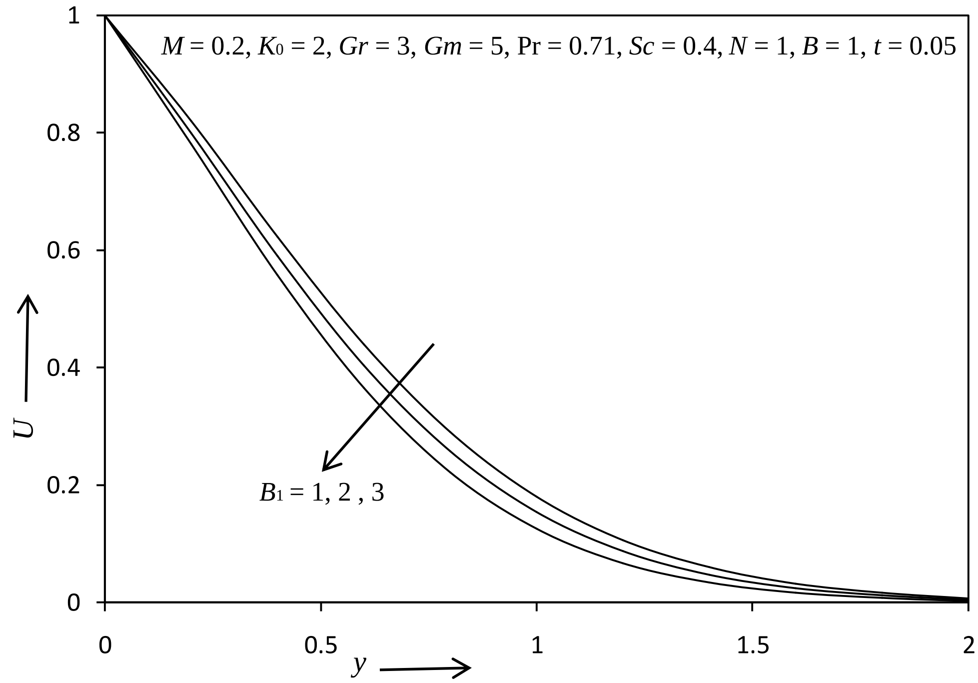

Fig: 8. The Velocity profile of dusty fluid for different value of *B*1

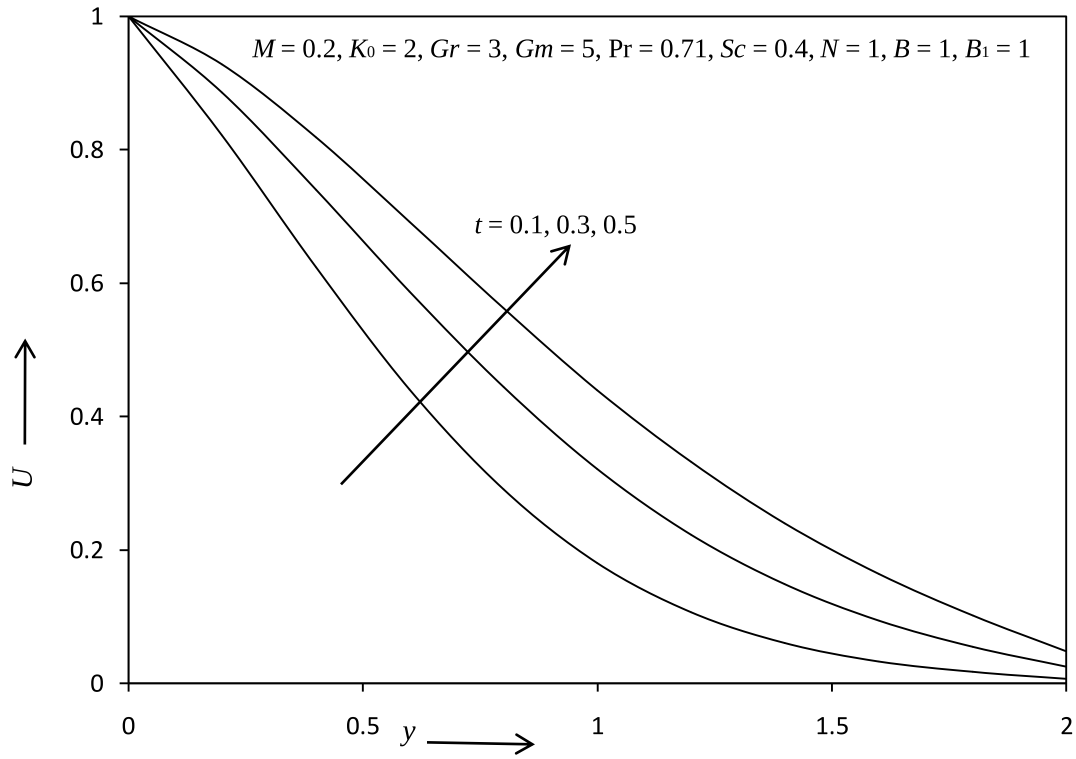

Fig: 9. The Velocity profile of dusty fluid for different value of *t*

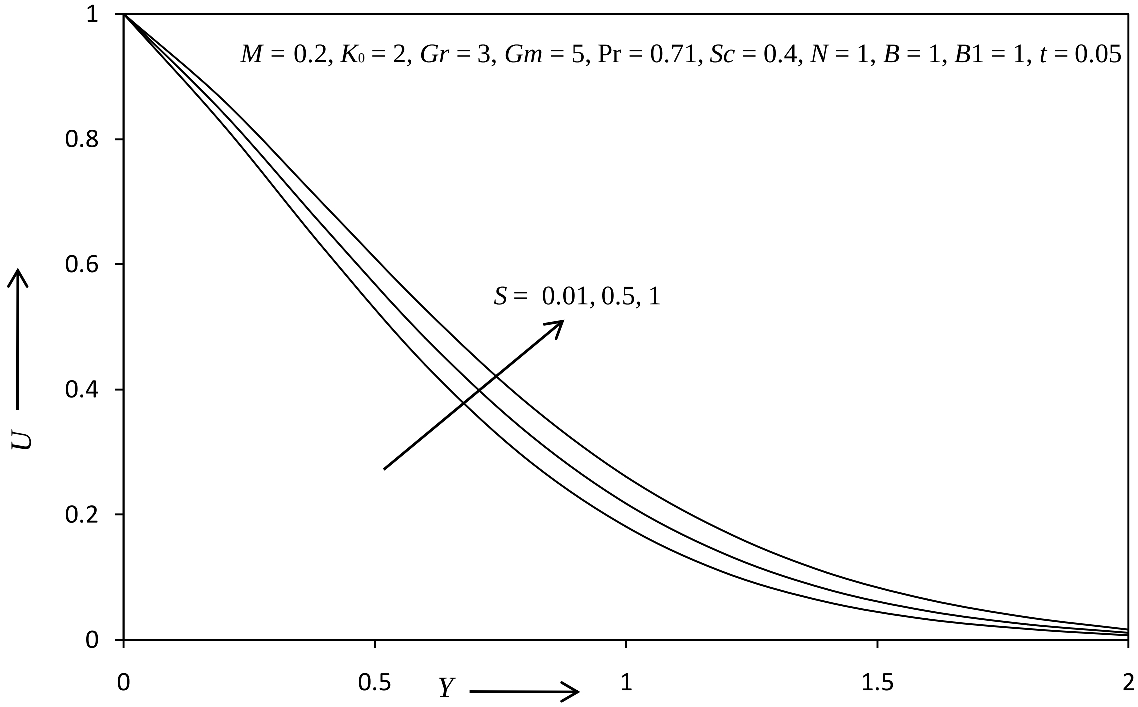

Fig: 10. The Velocity profile of dusty fluid for different value of S

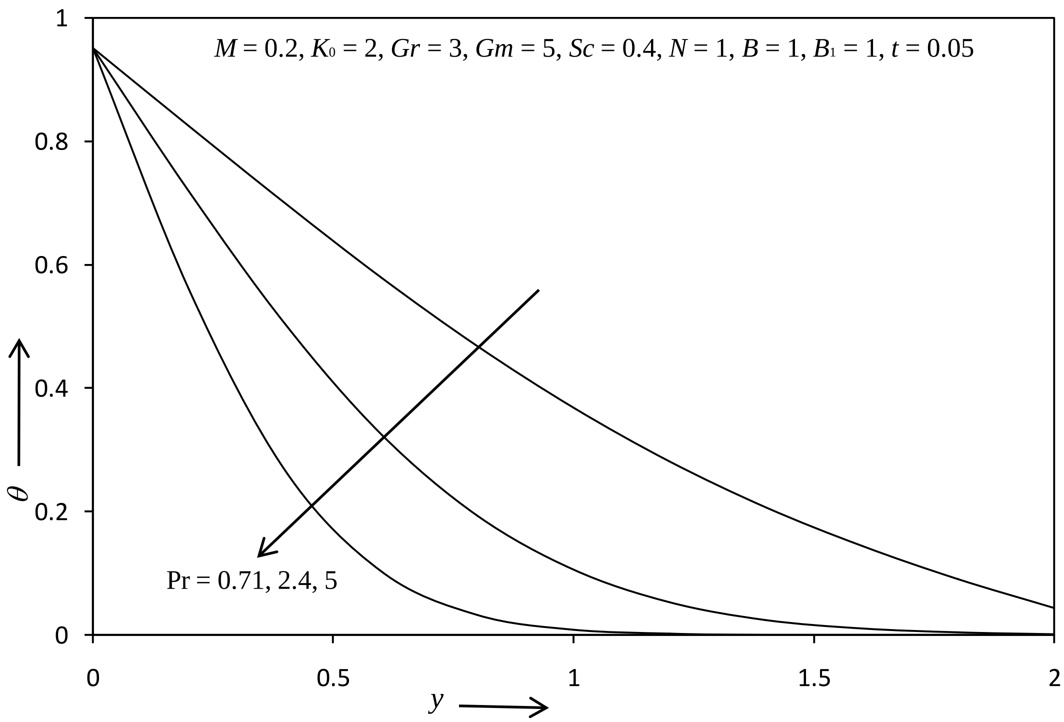

Fig: 11. The temperature profile of dusty fluid for different value of Pr

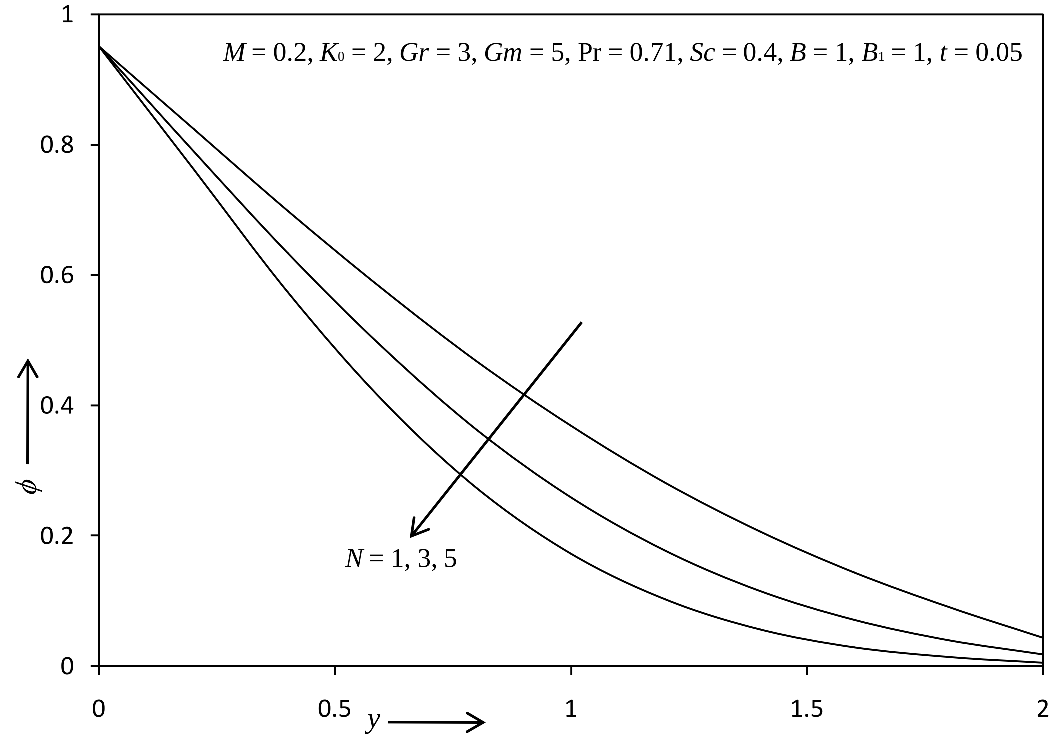

Fig: 12. The temperature profile of dusty fluid for different value of N

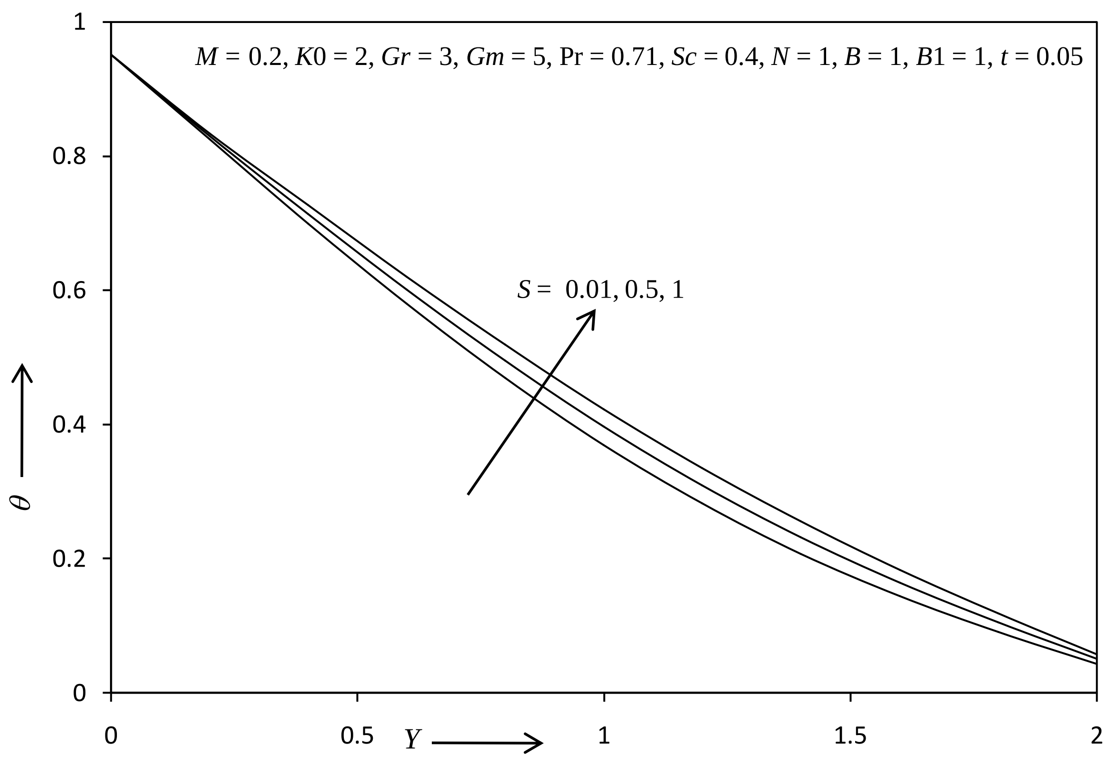

Fig:13. The temperature profile of dusty fluid for different value of S

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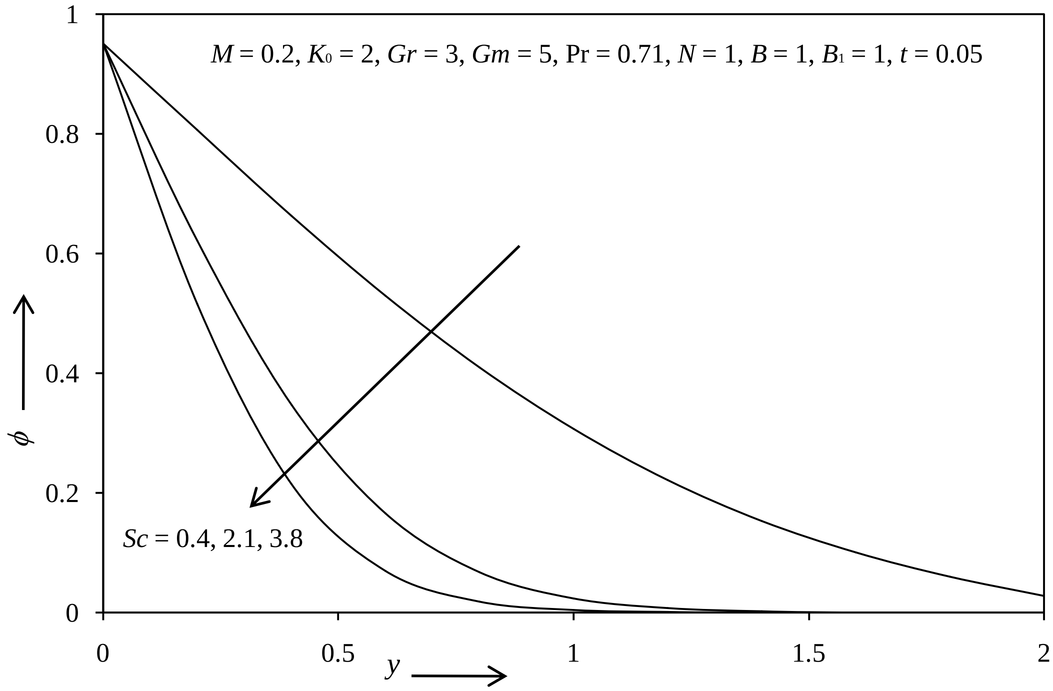

Fig:14. The concentration profile of dusty fluid for different value of *Sc*

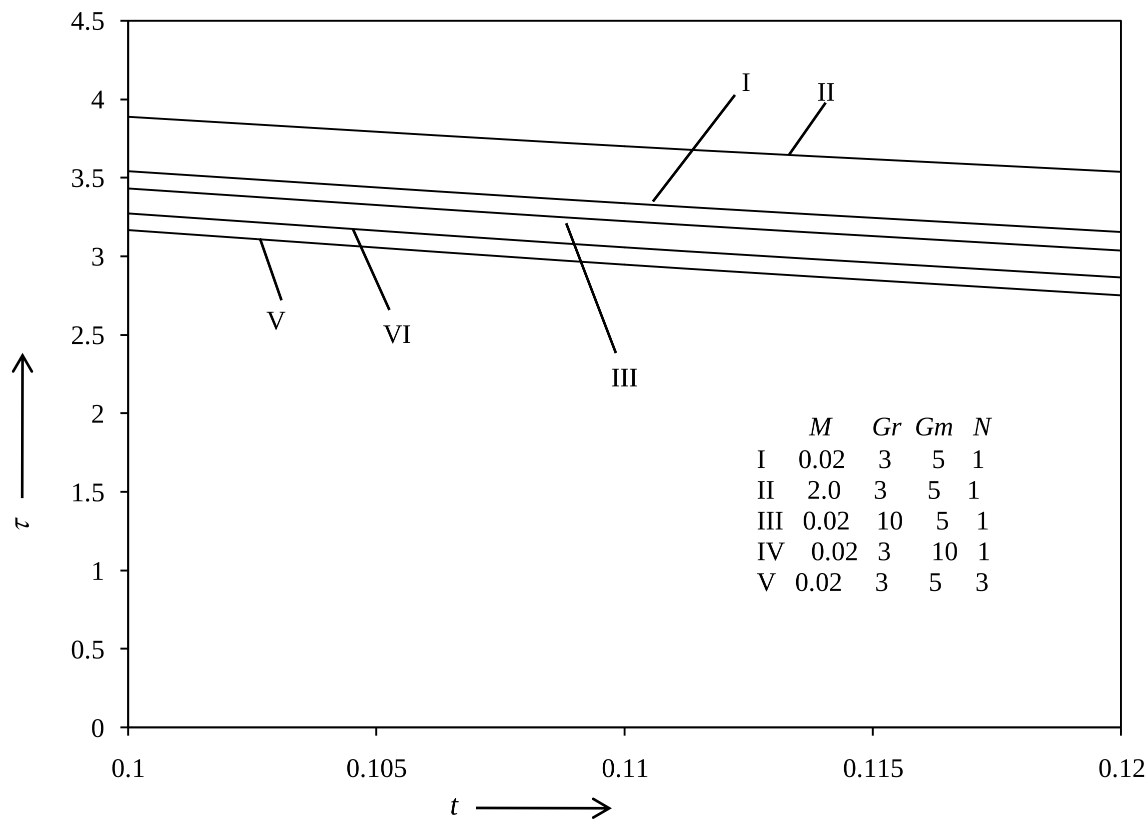

Fig: 15. Skin friction of dusty fluid for different value of *M*, *Gr*, *Gm* and *N*

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