Optimal Scheduling with Transportation Time and Equivalent Item for Item-Block

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Abstract

The present chapter is devoted to study a scheduling problem with two machines arranged in series with transportation time and with the concept of equivalent item for item block. In order to transport the processed job from first machine to second there is a single transport agent, which returns empty to first machine to carry next jog after processing. Further, an optimal solution has been obtained for the scheduling problem involving weighted items and breakdown interval. The proposed algorithm has been applied on a numerical problem.

1. Introduction

In this chapter we have discussed a more practical and realistic situation of scheduling. Earlier, while studying scheduling it was always assumed that the time of transporting an item from machine A and machine B is negligible, but if we look at it practically it is found that such an event is nearly impossible. Thus in this chapter we have considered that the transportation of i^{th} item from machine A to B is done by a transport agent. And the time taken by the agent in going from machine A to machine B and then returning back to A is counted. Besides, it the time spent by it in waiting for the completion of process on machine A is also counted as time lag.

Here, we have also considered that the items have to be processed in a particular group called "block" and in this block an item may be assigned priority over another item of the block. This blocking may cause an increase in the total production time and consequently, increase the production cost of the items. The concept of equivalent job for a job block has been introduced by Maggu and Das (1977). This idea is very useful to the useful to the industry for manufacturing items in groups. Thus we will study here a tandem queuing system with transportation time between the two machines. The restriction of item-block is introduces.

Statement of the Problem (P1)

Suppose we have n items (I_1, I_2... I_n) which are to be processed through two machines A and B in the order AB with a single transport agent who takes a single item from machine A to machine B. And then returns back empty to machine A to take the next item to machine B and so on until items are taken to B.

Let A_i, and B_i be respectively the times required to process the i^{th} item on two machines A and B. Also let t_i, and r_i be respectively the transportation time of item i from machine A to machine B and the returning time from machine B to machine A after delivering item i.

Consider the sequence S = (e_1, e_2... e_j, e_{j+1} ...en) where the items e_j and e_{j+1} must occur in this sequence as a block. We define a new item called equivalent item for the block (e_j, e_{j+1}) with processing times A_α and B_α on the two machines A and B respectively.

Let us define

$$R_{e_j} = \begin{cases} t_{e_j} + r_{e_j} - A_{e_{j+1}}, & \text{if } t_{e_j} + r_{e_j} > A_{e_{j+1}} \\ 0, & \text{if } t_{e_j} + r_{e_j} \le A_{e_{j+1}} \end{cases}$$

This is the time lag between the completion time of $(j+1)^{th}$ item on machine A and the returning time of transport agent after delivering item j at machine B.

The machine starts processing i^{th} item when $(i-1)^{th}$ is handed over to the transport agent. Thus R_{e_j} is the time lag of machine A. $A_{e_{j+1}}$ is the processing time of $(j+1)^{th}$ item on machine A but it will be considered complete i.e. $CA_{e_{j+1}}$ when it is handed to the transportation agent. Thus if the sum of transportation time from machine A to machine B and returning time from machine B to machine A is greater than the processing time then the machine has to wait and to what time it has to wait will be the time spent by it after completing the processing i.e. $t_j + r_j - A_{e_{j+1}}$. This is time lag of machine A after processing $(j+1)^{th}$ item. But if the processing of an item takes more time than $t_{e_j} + r_{e_j}$ then the time lag will be zero.

It can be observed that

$$A_\alpha = \Big[(R_{e_{j-1}} + t_{e_j} + A_{e_j}) + (R_{e_j} + t_{e_{j+1}} + A_{e_{j+1}}) - \min(R_{e_j} + t_{e_{j+1}} + A_{e_{j+1}}, R_{e_{j-1}} + t_{e_j} + A_{e_j}) \Big]$$

And

$$B_\alpha = \Big[(R_{e_{j-1}} + t_{e_j} + B_{e_j}) + (R_{e_j} + t_{e_{j+1}} + B_{e_{j+1}}) - \min(R_{e_j} + t_{e_{j+1}} + B_{e_{j+1}}, R_{e_{j-1}} + t_{e_j} + B_{e_j}) \Big] \quad ...(1)$$

Where

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$R_{e_{j-i}}$ = Time lag of machine A after completing processing of j[th] item.

t_{e_j} = Transportation time of j[th] item from machine A to B

A_{e_j} = Processing time of j[th] item on machine A

2. Development of Solution Procedure

Due to Maggu, Das and Kumar (1981), for the sequence S, we have

$$CB_{e_{j+2}} = \max\left[CA_{e_{j+2}} + t_{e_{j+2}} + R_{e_{j+1}}, CB_{e_{j+1}}\right] + B_{e_{j+2}}$$

$$= \max\left[CA_{e_{j+2}} + t_{e_{j+2}} + R_{e_{j+1}}, CA_{e_{j+1}} + t_{e_{j+1}} + R_{e_j} + B_{e_{j+1}},\right.$$

$$\left. CA_{e_j} + t_{e_j} + R_{e_{j-1}} + B_{e_j} + B_{e_{j+1}}, CB_{e_{j-1}} + B_{e_j} + B_{e_{j+1}}\right] + B_{e_{j+2}}$$

$$= \max\left[CA_{e_{j+2}} + t_{e_{j+2}} + R_{e_{j+1}}, CA_{e_{j+1}} + A_{e_{j+1}} + t_{e_{j+1}} + R_{e_j} + B_{e_{j+1}},\right.$$

$$\left[CA_{e_{j+2}} + t_{e_{j+2}} + R_{e_{j+1}}, CA_{e_{j+1}} + t_{e_{j+1}} + R_{e_j} + B_{e_{j+1}},\right.$$

$$\left. CA_{e_j} + t_{e_j} + R_{e_{j-1}} + B_{e_j} + B_{e_{j+1}}, CB_{e_{j-1}} + B_{e_j} + B_{e_{j+1}}\right] + B_{e_{j+2}}$$

The two middle terms combined may be written as follows–

$$= \max\left[CA_{e_j} + A_{e_{j+1}} + t_{e_{j+1}} + R_{e_j} + B_{e_{j+1}}, CAe_j + t_{e_j} + R_{e_{j-1}} + B_{e_j} + B_{e_{j+1}}\right]$$

$$= CA_{e_j} + \max\left[A_{e_{j+1}} + t_{e_{j+1}} + R_{e_j}, t_{e_j} + R_{e_{j-1}} + B_{e_j}\right] + B_{e_{j+1}}$$

The completion time of j+2 items on second machine may thus be written as–

$$CB_{e_{j+2}} = \max\left[CA_{e_{j+2}} + t_{e_{j+2}} + R_{e_{j+1}}, \max\left(A_{e_{j+1}} + t_{e_{j+1}} + R_{e_j}, B_{e_j} + t_{e_j} + R_{e_{j-1}}\right)\right.$$

$$\left. + B_{e_{j+1}}, CB_{e_{j-1}} + B_{e_j}\right] + B_{e_{j+2}}$$

...(2)

Now for any other sequence

$$S' = \left(e'_1, e'_2, e'_3 .... e'_{j-1}, \alpha, e'_{j+2}, ... e'_1\right)$$

$$C'B_{e_{j+2}} = \max\left[CA_\alpha + t_\alpha + R_{e_{j-1}}, CB_{e_{j-1}}\right] + B_\alpha$$

Thus

$$C'B_{e_{j+2}} = \max\left[C'A_{e_{j+2}} + t'_{e_{j+2}} + R'_{e_{j+1}}, C'B_\alpha\right] + B_{e_{j+2}}$$

$$= \max\left[C'A_{e_{j+2}} + t'_{e_{j+2}} + R'_{e_{j+1}}, C'A_\alpha + t_\alpha + R_{e_{j-1}} + B_\alpha, C'B_{e_{j-1}} + B_\alpha\right] + B'_{e_{j+2}}$$

...(3)

$CB_{e_{j+2}} = C'B_{e_{j+2}}$ as we have taken as a single term.

$$CA_{e_j} + \max\left[A_{e_{j+1}} + t_{e_{j+1}} + R_{e_j}, t_{e_j} + R_{e_{j-1}} + B_{e_j}\right] + B_{e_{j+1}}$$

$$= C'A_\alpha + t_\alpha + R_{e_{j-1}} + B_\alpha$$

Thus

$$CA_{e_j} + \max\left[A_{e_{j+1}} + t_{e_{j+1}} + R_{e_j}, t_{e_j} + R_{e_{j-1}} + B_{e_j}\right] + B_{e_{j+1}}$$

$$= C'A_\alpha + t_\alpha + R_{e_{j-1}} + B_{e_j} + B_{e_{j+1}}$$

Since

$CB_{e_{j-1}} = C'B_{e_{j-1}}$ and $CA_{e_{j-1}} = C'A_{e_{j-1}}$

We get from equations (2) and (3)

$$A_{e_j} + \max\left[A_{e_{j+1}} + t_{e_{j+1}} + R_{e_j}, t_{e_j} + R_{e_{j-1}} + B_{e_j}\right] + B_{e_{j+1}}$$

$$= A_\alpha + t_\alpha + B_{e_j} + B_{e_{j+1}}$$

Subtracting $B_{e_{j+1}}$ from both the sides and set $t_\alpha = 0$, we obtain

$$A_\alpha = A_{e_j} + \max\left[A_{e_{j+1}} + t_{e_{j+1}} + R_{e_j}, t_{e_j} + R_{e_{j-1}} + B_{e_j}\right] - B_{e_j}$$

...(4)

It is clear from equation (4) that the completion time of item j+2 on machine A for S differs by an amount equal to min from the sets.

The following theorem provides a method to obtain an optimal sequence.

3. Algorithm

Step: 1. Find new processing times for the equivalent item $\alpha = \left(e_j, e_{j+1}\right)$. Also find the transportation and returning times for each equivalent item.

Step: 2. Consider a new problem with processing times for the equivalent items. The processing times for the remaining items is as before.

Step: 3. Define two fictitious machines G and H with processing times Gi and Hi. Where $G_i = A_i + t_i + R_{i-1}$ and $H_i = B_i + t_i + R_{i-1}$

Step: 4. Apply Johnson's method to find the optimal sequence of the reduced problem in step 3.

Step: 5. Find the optimal sequence for alone ignoring the other items.

Step: 6. Replace each equivalent item by their ordered item block. Now this sequence gives optimal sequence for the original problem.

4. Numerical Illustration

Let a machine tandem queuing problem be given in the following tabular form-

Item (i)	Machine A (A_i)	t_i $\longrightarrow$	r_i $\longleftarrow$	R_{i-1}	Machine B (B_i)
1	5	3	3	-	7
2	7	4	3	0	4
3	4	3	3	3	8
4	7	5	3	0	3
5	8	6	3	0	9
6	6	3	3	3	5

(Times in hours)

Find the optimal sequence for the above problem where $\alpha = (2, 4, 6)$

Solution

Let $\eta = (2, 4)$ and $\alpha = (\eta, 6)$

We have

$$A_\eta = \left[A_{e_j} + t_{e_j} + R_{e_{j-1}}\right] + \left[t_{e_{j+1}} + A_{e_{j+1}} + R_{e_j}\right]$$

$$-\min\left[A_{e_{j+1}} + t_{e_{j+1}} + R_{e_j}, B_{e_j} + t_{e_j} + R_{e_{j-1}}\right]$$

$$A_\eta = 15$$

$$B_\eta = \left[B_{e_j} + t_{e_j} + R_{e_{j-1}}\right] + \left[t_{e_{j+1}} + B_{e_{j+1}} + R_{e_j}\right]$$

$$-\min\left[A_{e_{j+1}} + t_{e_{j+1}} + R_{e_j}, B_{e_j} + t_{e_j} + R_{e_{j-1}}\right]$$

$B_\eta = 8$ Also $t_\eta = R_\eta = 0$

Thus

$$A_\alpha = \left[A_\eta + t_\eta + R_\eta\right] + \left[t_6 + A_6 + R_6\right]$$

$$-\min\left[A_6 + t_6 + R_6, B_\eta + t_\eta + R_\eta\right]$$

$$A_\alpha = 19$$

$$B_\alpha = \left[B_\eta + t_\eta + R_\eta\right] + \left[t_6 + B_6 + R_6\right]$$

$$-\min\left[A_6 + t_6 + R_6, B_\eta + t_\eta + R_\eta\right]$$

$B_\alpha = 11$ Also $t_\alpha = R_{e_{j-1}} = 0$

Replace the item block (2, 4, 6) by the equivalent item then α, the reduced problem is

Item (*i*)	Machine *A* (A_i)	t_i →	r_i ←	R_{i-1}	Machine *B* (B_i)
1	5	3	3	0	7
A	19	0	0	0	11
3	4	3	3	3	8
5	8	6	3	0	9

Let G and H be two fictitious machines, then we find the new reduced problem as

Item (*i*)	Machine *G* (G_i)	Machine *H* (H_i)
1	8	10
A	19	11
3	10	14
5	14	15

Using Johnson's method, the optimal sequence of the above problem is: (1, 3, 5, α). The optimal sequence for is obtained as 6, 2, 4.

Therefore the optimal sequence for the original problem is

1	3	5	6	2	4

The total production time for the above optimal sequence is obtained in the following table:

Item	Machine A		t_i	r_i	Machine B		$Y = C_iA + t_i + r_i$	Ideal Time	
(*i*)	In	Out	→	←	In	Out		A	B
1	0	5	3	3	8	15	-	0	8
3	5	9	3	3	15	23	11	0	0
5	9	17	6	3	23	32	17	0	0
6	17	23	3	3	32	37	26	0	0
2	23	30	4	3	37	41	32	0	3
4	30	37	5	3	44	47	39	10	0
							Total	10	11

Where Y is the returning time of the transport agent from B to A. Idle time for machine A is 10 hours, for machine B it is 11 hours and for transport agent it is 5 hours. Now as variation in the above discussed problem (P_1), we will discuss the a two machine problem with a single transport agent involving item-block, weighted items and break-down times of machines.

5. Case of Weighted Items and Break-Down Interval of Machines (P_2)

Suppose there are n items (l_1, l_2...l_n) simultaneously available, each of which has to be processed in the same order by two given machines (A and B). A machine may not process more than one item at a time nor may an item be processed by more than one machine simultaneously, r_i is the returning time of the transport agent from machine B to machine A after delivering item *i*, and w_i is the weight of the ith item. Let be the equivalent item for the block (e_j, e_{j+1}) with the processing times (A_α, B_α) as defined in equation (1). Let the length of the break-down interval be given by L = b – a. A heuristic solution can be obtained by applying the following.

5.1. Algorithm for P_2

Step: 1. Use equation (1) to find new processing times for the equivalent item α = (e_j, e_{j+1}). Also find the transportation and returning times for each equivalent item.

Step: 2. Consider a new problem with processing times for the equivalent items as per step 1, i.e., replace the item-block by its equivalent item, the processing times for the remaining items are as before. The weight of the equivalent item is equal to the average of the weights of the items.

Step: 3. Define two fictitious machines G and H with processing times $G_i = A_i + t_i + R_i$ and $H_i = B_i + t_i + R_{i-1}$

Step: 4. Schedule the items according to the following rule: The items for which $G_i < H_i$; are processed in decreasing order according to their weights i.e., the highest weight item will be processed first and then next highest weight so on. The item for which $G_i \geq H_i$ are processed in a similar way as above, immediately after all items for which $G_i < H_i$ are processed.

Step: 5. Find the optimal sequence for alone ignoring the other items according to the rule in step 4.

Step: 6. Replace each equivalent item by their ordered item-block. Now this sequence gives optimal sequence for the original problem.

Step: 7. Find the flow-chart of the optimal sequence of the items of the problem in step (6) and read the effect of break-down intervals of machines on all items. Also find a new problem with the following processing times.

$X'_i = X_i$: if (*a*, *b*) has no effect on item *i*.

$= X_i + L$: if (*a*, *b*) has an effect on items *i*.

Where $X = G$ or H and L is length of the interval (a, b).

Step: 8. Obtain new processing times for the equivalent items as per the new problem in step (7), also find a new problem by replacing with its new processing times.

Step: 9. Find the optimal sequence for the new reduced problem in step (8) following the rule in step (4).

Step: 10. The optimal sequence in step (9) is the optimal sequence for the original problem.

Let us illustrate this procedure by the following numerical example.

6. Numerical Illustration

Suppose we were given the following data for a two machines in tandem.

Item (i)	Machine A (A_i)	t_i →	r_i ←	Machine B (B_i)	W_i
1	6	5	4	12	2
2	11	6	4	8	4
3	6	4	4	9	5
4	5	4	4	11	3
5	10	3	4	9	6

Where A_i, B_i, t_i, r_i and W_i are as defined before. Find the optimal sequence for the above problem where α = (2, 5) and $L = 23 - 18 = 5$.

Solution

In step (1) we have to calculate

$$R_{i-1} = t_{i-1} + r_{i-1} - A_i \quad , \quad if \quad t_{i-1} + r_{i-1} > A$$

$$= 0 \quad , \quad \text{otherwise}$$

We have given α = (2, 5). Using equation (1), we have $A_\alpha = 27$, $B_\alpha = 13$.

As per steps (1) and (2) we have to replace the item block (2, 5) by the equivalent item then the new problem is

Item (i)	Machine A (A_i)	t_i →	r_i ←	Machine B (B_i)	W_i
α	27	0	0	13	5
1	6	5	0	12	2
3	6	4	4	9	5
4	5	4	0	11	3

The new reduced problem of step (3) is

Item (i)	Machine G (G_i)	Machine H (H_i)	W_i
α	27	13	5
1	11	17	2
3	14	17	5
4	9	15	3

As per step (4) the optimal schedule of the above problem is 1, 3, 4 or 1, 2, 5, 3, 4. According to steps (5) and (6) the optimal schedule of the items is 1, 5, 2, 3, 4.

The calculations of the total production time are done in the following table:

Item	Machine A		t_i	r_i	Machine B		$Y = C_iA + t_i + r_i$	Ideal Time	
(i)	In	Out	→	←	In	Out		A	B
1	0	6	5	4	11	23	-	0	11
5	6	16	3	4	23	32	15	0	0
2	16	27	6	4	33	41	22	0	1
3	27	33	4	4	41	50	32	0	3
4	33	38	4	4	50	61	40	23	0
							Total	23	12

As per step (7), the new processing times are given in the following tabular form.

Item (i)	Machine G (G_i)	Machine H (H_i)	W_i
1	11	22	2
2	22	14	4
3	14	17	5
4	9	15	3
5	17	16	6

By step (8), the new processing times for the equivalent items are –

$A\alpha = (22+17) - \min(17, 14) = 39 - 14 = 25$

$B\alpha = (14+16) - \min(17, 14) = 30 - 14 = 26$

Replacing by its new processing times, we have

Item (i)	Machine G (G_i)	Machine H (H_i)	W_i
α	25	26	5
1	11	22	2
3	14	17	5
4	9	15	3

The optimal sequence of the problem after we take into account the break-down times of machine is: 1, α, 3, 4 or 1, 5, 2, 3, 4 which is the same as when break-down of machines is not taken into account.

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