A Stochastic Model for HIV Patients Using Three Parameters Exponential Weibull Distribution

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Abstract

The present research study attempts to developed a mathematical model for calculating expected time of break down point/servo conversion threshold level using exponential weibull distribution which would helps reduce the spread of HIV from once person to another by models such as needle sharing among drug abusers, prostitutes, homosexuals and blood transfusion and so as such that more accurate data could be obtained about the accurate data could be obtained about the given population of study, thus decreasing the spread as well as pointing out the mutual independence of inter arrival time between successive contacts sequence damage and threshold.

1. Introduction

The present researchers purpose to allow how the external infectiousness of semen of the HIV infected patient differs at each stage of the disease. The actual infectiousness varies from to the other hence the present research plans to compute an average. Further, the extent of infectiousness depends on the type of sexual; the higher for some types compared to the others. A mathematical model is obtained for the expected time of the breakdown point to each the servo conversion threshold level. In the servo conversion of HIV/AIDS, the assumptions that time between contact periods are independently identically distribution random variable. The Exponentiated Weibull distribution family (EWD) was initially proposed by Mudholkar and Srivastava (1993). It covers the one-parameter exponential family. Exponentiated exponential family as a sub-family as well as the most properly used two parameter Weibull family as a special sub-family.

EWD family allows non-monotonic hazard functions. This is one of its unique features. In the order to leaves further details about the expected time to cross the threshold level of sero conversion period one can refer to Esary *et.al.,* (1973), Elangovan *et.al.,* (2009), Pandiyan *et.al.,* (2015).

The factors responsible for the damage after the immune system is effectual are ELISA- HBE +VE, ELISA- HBE –VE, WESTERN TEST, CD4+T CELL TEST and so an. The present research makes an attempt to fit original data in the model derived for the Expected time. The observed data has been derived from Cuddalore region.

2. Assumptions of the Model

- Sexual contacts are the only source of HIV infection person.
- The threshold of any individual is a random variable.
- If the total damage crosses a threshold level Y which itself is a random variable, the sero conversion occurs and a person is recognized as an infected.
- The inter arrival time between the successive contacts are at random variable which are identically.

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3. Model Description

The Cumulative density function (CDF) of the three parameter Exponentiated Weibull distribution

$$F(x,\lambda) = \left[1 - e^{-\left(\frac{x}{\lambda}\right)^k}\right]^\alpha ; \qquad x > 0$$

The corresponding survival function is

$$\overline{\mathrm{H}} = \mathrm{e}^{-\left(\frac{\mathrm{x}}{\lambda}\right)^{\mathrm{k}}} \qquad (1)$$

Y: Continuous random variable denoting the threshold level of three parameter Exponentiated Weibull distribution.

$$P(X_i < Y) = \int_0^\infty g^*(x)\overline{H}(x)dx \qquad (2)$$

S(t) : the survivor function i.e $P(T > t)$

$$P(T > t) = \sum_{k=0}^\infty V_k(t)P(X_i < Y) \qquad (3)$$

Taking Laplace Transformation of the life time $= 1 - S(t)$ we get Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (4) we get

$$l^*(s) = \frac{\left[1 - g^*\left(\frac{1}{\sigma}\right)^\beta\right] f^*(s)}{\left[1 - g^*\left(\frac{1}{\sigma}\right)^\beta f^*(s)\right]} \qquad (4)$$

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$$= \frac{\left[1 - g^*\left(\frac{1}{\sigma}\right)^\beta\right]\left(\frac{c}{c+s}\right)}{\left[1 - g^*\left(\frac{1}{\sigma}\right)^\beta\left(\frac{c}{c+s}\right)\right]} \qquad \ldots(5)$$

$$E(T) = -\frac{d}{ds} l^*(s) \quad given \quad s = 0$$

$$= \frac{1}{c\left[1 - g^*\left(\frac{1}{\lambda}\right)^\beta\right]} \quad \ldots (6)$$

Then

$$E(T) = \frac{\mu\lambda^k + 1}{c} \quad \ldots (7)$$

Where

c=Time interval

μ = CD4 counts cell

λ= Viral RNA

k= Total leucocyted count

Table 1: Data observed of the infected

C Time interval	μ CD4 counts cell	λViral RNA	k Total leucocytes count
1	870	2780	3921
2	773	2130	3332
3	614	1970	2236
4	522	1321	1780
5	489	1022	1541
6	360	972	1312
7	310	822	1113
8	220	613	995
9	210	393	847
10	216	204	736

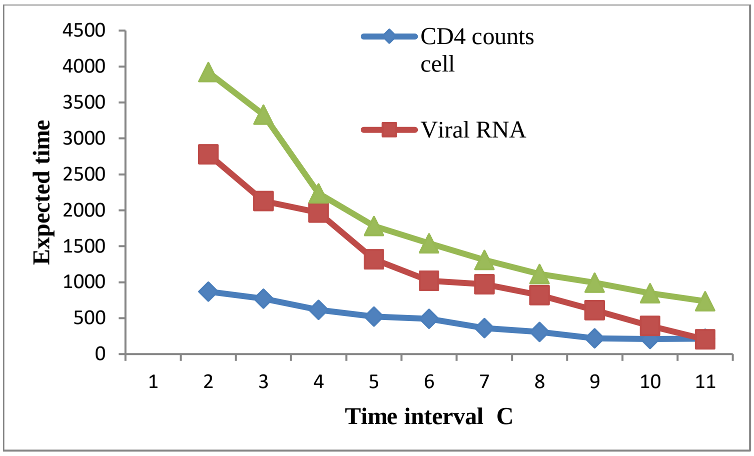

Fig. 1 Blood count with time

4. Conclusions

The present researcher based on the above study arrives at the conclusion that a person infected with Human Immune virus cross the threshold level more quickly. Once a person in infected the immune cell are damaged and that he/she is likely to be infected when they are infected by the immune virus. The time interval duration for the infection depends on the period of sexual contact of the infected person. There is an abrupt lowering to the threshold level of the expected life spam of the person. The modeling clearly observes that once the person is infected with HIV, the immune system gets damaged. The model also shows that in the case of an he infected person the breakdown of the immune system starts . If the number of sexual contact increases after the threshold level then the expected life time of HIV the patient will decrease, in as observed in the above table and figure. The life span of the affected person can be prolonged by proper medical advice and through regular medication.

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