

Optimization of helical gear design using Genetic Algorithm for center distance

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Abstract

The helical gears are employed to transmit motion between parallel shafts. These gears can also be used for transmitting motion between non-parallel, non-intersecting shafts. Helical gears are similar to spur gears except that the gears teeth are at an angle with the axis of the gears. In a genetic algorithm, populations of individuals, which are potential solutions to the optimization problem, undergo a sequence of unary and higher order transformations (also known as mutations and crossovers respectively). Genetic algorithms are being widely used for optimization, search and neural network synthesis. A lot of work has been conducted using genetic algorithm for different engineering problems related to scheduling and process planning in industrial engineering, network optimization in computer engineering. The work dissertation deals with the optimization of helical gear sets using genetic algorithm. Attention is focused on reducing the center to center distance of gear set subjected to constraints on bending stress, contact stress and involute interference.

1. Introduction

Gear design is a complex phenomenon requiring consideration of several items such as gear geometry, material heat treatment, manufacturing, etc., to satisfy. Functional requirement, of high strength, high accuracy, low noise, and compactness of the drive. Traditionally, gear designers have been concerned with requirements of strength, noise, life, and accuracy of kinematic transmission. The recent focus of research however is the optimal design of compact gear pairs (gear boxes) for minimum weight and space requirements. Savage et al [3] were probably the first to attempt the optimal design of compact spur gear sets using the American gear Manufactures Association (AGMA) standards. Subsequently Carroll and Johnson expanded this work by using the technique for computation of 'J' factor reported by Mitchiner and Mabie. The objective function to be minimized was the center distance of helical gear pair subject to constraints on bending stresses, contact stresses and involutes interference. Standard gear tooth geometry was considered. No work has been reported, to our knowledge, on the design of compact helical gear sets [15]. A helical gear is termed right handed or left handed as determined by the direction the teeth slope away from the viewer looking at the top gear surface along the axis of the gear. Alternatively if a gear rests on its face the hand is in the direction of the slope of the teeth. Meshing helical gears must be of opposite hand. Meshed helical gears can be at an angle to each other (up to 90°). The helical gear provides a smoother mesh and can be operated at greater speeds than a straight spur gear. In operation helical gears generate axial shaft forces in addition to the radial shaft force generated by normal spur gears. In operation the

initial tooth contact of a helical gear is a point which develops into a full line contact as the gear rotates. This is a smoother cycle than a spur gear which has an initial line contact. Spur gears are generally not run at peripheral speed of more than 10m/s. helical gears can be run at speed exceeding 50m/s when accurately machined and balanced. Helical gears are circular gears whose teeth curve along a helical path. Helical-toothed gears transmit power and motion between parallel axes (opposite hand) or right-angle axes (same-hand). Helical gears may be anti-backlash, meaning they have a mechanical assist such as a spring to take up any play between meshing gear teeth, thus avoiding backlash when gear direction changes. Important parameters to consider when specifying helical gears include number of teeth, pitch diameter, face or tooth width, and outside diameter. The number of teeth, along with desired pitch diameter, will dictate the pitch of the gear. The face width is width of the teeth. Gear hand direction and pressure angle are also important to consider. Helical gears have a right or left hand specification, which describes the direction of tooth curve. Helical gears of opposite hand (one right and one left) will mesh for parallel-axis power transmission. Same-hand pairs will mesh for perpendicular or off-axis power transmission, depending on the helix angle. Gears must have the same pressure angle to mesh [22]. Other important specifications to consider for helical gears include materials of construction and mounting. Materials of construction can be metal or plastic.

The objectives of present work are:

- (1) The design of helical gear set for the given constraint on contact stress, bending stress and involutes interference using genetic algorithm.
- (2) To find the variation of stresses developed in the gear with pinion number of teeth, helix angle and diameter pitch.

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In general, the most desirable gear set is the smallest one that will perform the required job. Smaller gears are easier to make, run more smoothly due to smaller inertial loads and pitch line velocities, and are less expensive. Previous research presented different approaches for optimum gear design. Jog and Pande optimized the gear design by minimizing the center distance of gear pair [10]. They considered involute interference, contact stress, and bending fatigue in their design. Carrol and Jhonson expanded the model to include the AGMA geometry factor and AGMA dynamic factor in the tooth strength formulas. Lin et al. developed a procedure to design compact helical gear sets including dynamic consideration. Rather than using the AGMA dynamic factor for medium accuracy gears, this increases as a simple function of pitch line velocity. And bending strength can be separated from the rest of the constraints which are of a geometrical nature. The evaluation of the capacity group is complex and time consuming, whereas that of the geometry group is relatively simple.

1.1 Problem Formulation: Parameters considered in the design of a helical gear pair include:

- Number of teeth on the pinion
- Diametral pitch
- Pressure angle
- Helix angle
- Face width of gear
- Material properties

Generally a normal pressure angle of 20 deg is used. The material properties are assumed to be constant once the materials and their heat treatments are specified (stainless steel, bronze and nylon). Similarly, the face width of a helical gear is chosen to be a function of ($\lambda \geq 2.0$) to get the benefit of the helical action. In view of the above consideration, the optimization problem with the Three design variables [10]:

- Number of teeth on the pinion (N_1)
- Diametral pitch (d_p)
- Helix angle (ψ)

In the work presented here the design space vectors for helical gear is identified by taking variables as diametral pitch, number of teeth of pinion and helix angle. It is, therefore, thought appropriate to consider the volume (or weight) of the pinion as the objective function which is to be minimized instead of the center-to-center distance of helical pairs. Constraints in the problem are involute interference, bending stress and contact stress at initial and final point of contact. In the approach here AGMA dynamic factor for high precision gears is taken in to consideration to account the dynamic effect. A complete program of genetic algorithm has been developed in C language to solve the problem in this program population size is 50, single point crossover is used with a probability of 0.6, probability of mutation is 0.016 and stopping criteria is given as 50 number of generation. By running the program, value of diametral pitch, helix angle

and number of teeth of pinion can be found at which the volume is optimum and all the constraints are satisfied [10].

2. Approach

The materials used as metallic or non-metallic for the optimization of helical gear is design. The genetic algorithm is used to accomplish this task because GA is better than conventional method. Genetic algorithm is based on the evolutionary ideas of natural selection and genetics. The objective function and constraints are described and GA implementing procedure is discussed. To a large extent, the satisfactory performance of a gear is a function of the material it is made of. Hence a proper selection of material and heat treatment is a prerequisite for efficient gear design. To achieve surface hardness in the range 555 to 627 HB, the most common practice is to case carburize the teeth from low carbon steel. The core hardness of such teeth after carburization lies in the range 255 to 375 HB [3].

3. Helical Gear

The teeth on helical gears are cut at an angle to the face of the gear. When two teeth on a helical gear system engage, the contact starts at one end of the tooth and gradually spreads as the gears rotate. Two mating helical gears must have equal but opposite helix angle. They have higher load capacity, are more expensive to manufacture. Helical gears can be used to mesh two shafts that are not parallel and can also be used in a crossed gear mesh connecting two perpendicular shafts. They have longer and strong teeth. They can carry heavy load because of the greater surface contact with the teeth.

1. Those connecting parallel shafts
2. Those connecting nonparallel shafts



Fig. 1 Right hand helical gear [20]



Fig.2. Left hand helical gear [20]

4. Objective Function

The mathematical or objective function is one wants to maximize or minimize, subject to certain constraints. Many optimization problems have a single objective function. (When they don't they can often be reformulated so that they do) The two exceptions are:

- 1. No objective function:** In some cases (for example, design of integrated circuit layouts), the goal is to find a set of variables that satisfies the constraints of the model. The user does not particularly want to optimize anything and so there is no reason to define an objective function. This type of problems is usually called a feasibility problem.
- 2. Multiple objective functions:** In some cases, the user may like to optimize a number of different objectives concurrently. For instance, in the optimal design of panel of a door or window, it would be good to minimize weight and maximize strength simultaneously. Usually, the different objectives are not compatible; the variables that optimize one objective may be far from optimal for the others. In practice, problems with multiple objectives are reformulated as single-objective problems by either forming a weighted combination of the different objectives or by treating some of the objectives as constraints.

Along with the electronic computer came the growth of optimization theory (the mathematics of achieving the best) applied to design problem. Optimization theory would require formulation of objective function that precisely measures cost or profit, and the expression of all side condition as mathematical equation or inequalities. The combination of design variable, giving the best possible value of the objective, which is consistent with constraints, is then sought by certain optimization procedures [17]. The design objective of this study is to obtain the most compact helical gear set satisfying design requirements that include power level, gear ratio and material parameters. The gears design must satisfy operational constraints including avoiding interference, pitting stress, scoring stress and bending stress. The required expressions for face width (F), pitch radius of the pinion (R₁), and the central-to-center distance (C) are chosen parameter to be optimized. The material properties are assumed to be constant once the materials and their heat treatments are specified. Similarly, the face width of a helical gear is chosen to be a function of λ (λ ≥ 2.0) to get the benefit of the helical action [10].

$$F = \frac{(\lambda \times \pi \times M_n)}{\sin \psi}, \text{ where } M_n = \text{normal module, } \psi = \text{helix angle} \tag{1}$$

$$R_1 = \frac{M_n \times N_1}{2 \cos \psi}, \text{ where, } N_1 = \text{number of teeth on pinion} \tag{2}$$

$$C = \frac{M_n \times N_1 (1 + m_g)}{2 \cos \psi}, \text{ where } m_g = \text{Gear ratio}$$

$$M_n = M_t \cos \psi, \text{ where, } M_t = \text{Transverse module}$$

$$M_n = \frac{d_p}{N_1} \times \cos \psi, \text{ where, } d_p = \text{diametral pitch} \tag{3}$$

As can be seen from these equations, a decrease in the helix angle decreases the center distance at the expense of an increase in face width. It is, therefore, thought appropriate to consider the volume of the pinion as the objective

function to be minimized instead of the center-to-center distance.

Considering the pinion to be a cylinder of radius R₁ and face width F, volume is

$$V = R_1^2 F \tag{4}$$

Substituting for F and R₁ from (1) and (2) in equation (4) and simplifying, the objective function becomes Volume parameter

$$Z = \frac{\lambda \times N_1^2 \times M_n^3}{\sin \psi \times \cos^2 \psi} \tag{5}$$

Putting the value of M_n from equation 3 in equation 5 and we get

$$Z = \frac{\lambda \times d_p^3 \times \cot \psi}{N_1} \tag{6}$$

Table 1 Basic gear design parameters and variables

Gear parameter	Design variables
1. Bending strength and contact strength limits	1. Number of Pinion teeth
2. Gear ratio	2. Diametral pitch
3. Face width	3. Helix angle
4. Pressure angle	
5. Addendum/Dedendum ratio	

5. Genetic Algorithms

The basic purpose of genetic algorithms (GAs) is optimization. Since optimization problems arise frequently, this makes GAs quite useful for a great variety of tasks. As in all optimization problems, we are faced with the problem of maximizing or minimizing an objective function f(x) over a given space X of arbitrary dimension. A brute force which would consist in examining every possible x in X in order to determine the element for which f is optimal is clearly infeasible.

Table 2 Comparison of natural and GA terminology

Sl.No.	Natural	Genetic Algorithm
1.	Locus	String position
2.	Gene (eye colour)	Characters, feature
3.	Chromosome	String
4.	Genotype	Structure (set of strings)
5.	Allele (blue)	Feature value
6.	Phenotype	Alternative solution

6. Genetic Algorithm Operation

The steps for applying GA are [7]:

1. Modeling
2. Choose a fitness function
3. Choose operators
- 3.1** Reproduction
- 3.2** Crossover
- 3.3** Mutation
4. Choose parameters
5. Choose initialization and stopping criteria

7. Results and Discussions

Based on the concepts of the Genetic Algorithm as present in previous, a complete program in C language has been developed for optimization of center distance of helical

gear set. By using this program the string combination from the search area with the maximum fitness and satisfying the given constraints is obtained. The pinion is taken as the optimization element following the trend of gear design due to its advantages. The problem of helical gear set was also solved by Jog and Pande [10]. In this chapter results obtained by GAs are discussed and compared with the results of Jog and Pande in tabular as well as in graphical form. In the helical gear design optimization problem center distance of helical gear set is prime consideration; it depends largely upon diametral pitch, helix angle and number of teeth of pinion it is also affected by the properties of the gear material. In the genetic algorithm program following parameters are used population size is 50, single point crossover is used with a probability of 0.6, probability of mutation is 0.016 and stopping criteria is given as 50 number of generation.

Table 3 Basic starting design input parameters of sample gear set

1. Diametral pitch (mm)	2.0
2. Pinion teeth (number)	21
3. Gear teeth (number)	84
4. Pressure Angle (degree)	20
5. Addendum ratio	1.286
6. Dedendum ratio	0.714
7. Helix angle (ψ)	10
8. Gear ratio	4.0
9. Contact ratio	1.292
10. Dynamic factor (K_v)	0.836

Table 3 shows the basic gear parameters for a sample gear set being studied. They were used in a gear design by Jog and Pande [10]. Jog and Pande worked on a sample gear set and obtained the optimized volume (center distance) as 172109.7 mm³. The optimum gear set designed by genetic algorithm has reduced volume (center distance) by 164873.1mm³as compared to Jog and Pande sample gear set volume (center to center distance). In other words, a better life gear design is found. The range of the variables is described in the table shown below.

Table 4 Range of variables

No.	Parameters	Ranges
1.	Diametral pitch	Range (2.0 - 20)
2.	Number of teeth	Range (10 - 206)
3.	Helix Angle	Range (10 – 35)

In the program, input design parameters are taken as shown in table 1.4. A complete program of genetic algorithm has been developed in C language to solve the problem. By running the program, value of volume gear set was found and all the constraints were satisfied at the maximum fitness function value. A criterion for the optimization is fitness value, the point at which fitness value is maximum are the optimum points. After running the program hundred

generations were developed, in each generation one optimum result was printed. The optimum result refers to the string combination at which the value of fitness was maximum in that particular generation of strings after the application of the operators of crossover and mutation. The new generation is produced and the best string in that generation is obtained. From these hundred generations the intermediate generation of string along with their fitness is shown as below in the Table 4.3. optimum results are shown in bold. It is clearly shown in Table 4.3 that maximum fitness is 0.086974, at which number of pinion teeth are 67, diametral pitch 12 and helix angle 25.

Table 1.5 Comparison of results obtained by GA with Jog and Pande [10] for sample Gear set

Optimization approach	Dia. pitch	No. of teeth	Helix angle (deg.)	(Center distance) Volume (mm ³)× 10 ⁴	Bending stress (MPa)	Contact stress (MPa)
Jog C.S. and Pande S.S.method	10.0	62	28	17.210970	154.2	521.7
Genetic Algorithm (Stochastic)	12	67	25	16.487315	117.0	391.5

In Table 1.5 gives comparison of results obtained from Jog and Pande [10] and genetic algorithm. Jog and Pande studied the sample gear set the found optimum (center distance) volume 172109.7 mm³. The optimum gear set using genetic algorithm has a (center distance) volume of 164873.15 mm³. In other words, a more compact design was found. Compared to Jog and Pande results, bending stress has decrease from 154.2 MPa to 117.0 MPa and there is also decrease in contact stress from 521.7 MPa to 391.5 MPa.

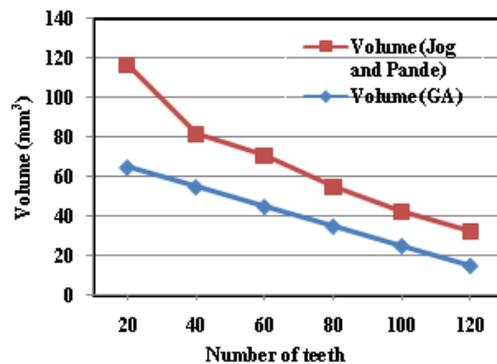


Fig.3 Variation of volume with number of teeth

Figure: 3 shows the variation of volume with pinion number of teeth. In each variation diametral pitch and helix angle are kept constant. At optimum design diametral pitch is 12 mm, number of teeth is 67 and helix angle are 25 deg. It means that volume or center distance is decreased and numbers of teeth are increased. It is clear from the figure that the number of teeth in case of the optimum volume or center distance given by the genetic algorithm method is found to be less than center distance in terms of volume corresponding to Jog and Pande. The center distance in terms of volume in the previous work of Jog and Pande was 17210970 mm³. In case of gear designed by GA center distance in terms of volume is 16487315 mm³. Conclusively it can be said that for the same center distance in terms of volume, the gear set designed by GA has lower volume as compared to that derived by Jog and Pande. A compact gear set have been obtained.

Conclusions

Genetic Algorithm can be used for the study of the problem for further extended in the following directions.

1. In the present work gear center-to-center distance is considered to be optimized by using number of teeth of pinion, diametral pitch and helix angle as design variable and volume of gear can also be optimized.
2. Misalignment's sensitivity can be determined.
3. By modifying GA's operator i.e. reproduction and crossover advanced genetic algorithm can be developed with the intention of getting better optimization.
4. Other materials can be chosen for the optimum designing of helical gear set, while keeping the same objective function and design constraints as taken in genetic algorithm technique.

The genetic adaptive search can be applied to the sample gear set and results can be compared with genetic algorithms.

Using Genetic Algorithm in the helical gear design and optimize the center distance of helical gear set is prime consideration; it depends largely upon diametral pitch, helix angle and number of teeth of pinion it is also affected by the properties of the gear material. In the genetic algorithm program in C language and the following parameters are used for population size is 50, single point crossover is used with a probability of 0.6, probability of mutation is 0.016 and stopping criteria is given as 50 number of generations. On running program for rpm=820 and power=100 hp following conclusion can be drawn.

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