

Radiation and Heat Absorption Effects on Unsteady MHD Flow Through Porous Medium in The Presence of Chemical Reaction of First Order

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Article Info

Article history:

Received 29 December 2013

Received in revised form

10 January 2014

Accepted 20 January 2014

Available online 1 February 2014

Keywords

Gray Radiation

Magnetic Field

Porosity Parameter

Chemical Reaction

Heat Absorption

Vertical plate

Abstract

In this paper the numerical solution of thermal radiation and heat absorption effects on unsteady MHD free convective flow through porous medium over a moving vertical plate in the presence of chemical reaction of first order is studied. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering porous medium. The temperature as well as concentration is raised linearly with respect to time. The dimensionless governing equations are solved using the finite difference technique. The velocity, temperature and concentration profile are discussed graphically for different parameters like the magnetic field parameter, porosity parameter, radiation parameter, chemical reaction parameter and heat absorption.

1. Introduction

Magneto convection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al. [8].

MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al. [9]. The dimensionless governing equations were solved using Laplace transform technique. Radiative convective flow are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from

large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. England and emery [5] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar [7] have considered the radiative free convective flow of an optically thin gray gas past a semi-infinite vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar [6] in all above studies, the stationary vertical plate is considered. Das et al. [3] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique. Diffusion rates can be altered tremendously by chemical reactions. The effects of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. Chamber and young [2] have analyzed a first

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order chemical reaction in the neighborhood of a horizontal plate. Apelblat [1] studied analytical solution for mass with a chemical reaction of first order. Das et al. [4] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. The dimensionless governing equations were solved by the usual Laplace-transform technique. Recently, Muthucumaraswamy et al. [10] have discussed on Radiation and MHD effects on moving infinite vertical plate in the presence of chemical reaction of first order solved by the usual Laplace-transform technique.

It is proposed to study thermal radiation and heat absorption effects on unsteady MHD flow through porous medium over past an impulsively started infinite moving vertical plate with variable temperature and mass diffusion in the presence of transverse applied magnetic field and first order chemical reaction. The governing equations are solved by the finite difference technique. The effect of velocity, temperature and concentration for different magnetic field parameter, porosity parameter, heat source parameter, chemical reaction parameter, radiation parameter and time are studied graphically.

2. Mathematical Analysis

Thermal radiation and heat absorption effects on unsteady MHD flow through porous medium over past an impulsively started infinite vertical plate with variable temperature and mass diffusion, in the presence of chemical reaction of first order. The z-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration. At time t , the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity v in a fluid, in the presence of thermal radiation. At the same time, the plate temperature as well as plate concentration are raised linearly with time. A transverse magnetic field of uniform strength is assumed to applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. It is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering porous medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - \frac{v}{K'}u \quad \dots (1)$$

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + Q(T - T_\infty) \quad \dots (2)$$

$$\frac{\partial C}{\partial t'} = D \frac{\partial^2 C}{\partial y^2} - K_1 C \quad \dots (3)$$

In the most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the order n , if the reaction rate is proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

With the following initial and boundary conditions:

$$\left. \begin{aligned} t' \leq 0 : u = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{for all } y \\ t' > 0 : u = u_0, \quad T = T_\infty + (T_w - T_\infty)At', \quad C = C_\infty + (C_w - C_\infty)At' \quad \text{at } y = 0 \\ : u = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \dots (4)$$

Where $A = \frac{u_0^2}{\nu}$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma (T_\infty^4 - T^4) \quad \dots (5)$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad \dots (6)$$

By using equations (5) and (6), equation (2) reduced to

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{16a^* \sigma T_\infty^3}{\rho C_p} (T_\infty - T) + Q(T - T_\infty) \quad \dots (7)$$

On introducing the following non-dimensional quantities:

$$\left. \begin{aligned} U = \frac{u}{u_0}, \quad t = \frac{t' u_0}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \\ Gr = \frac{g \beta \nu (T_w - T_\infty)}{u_0^3}, \quad Gc = \frac{g \beta^* \nu (C_\infty - C_w)}{u_0^3}, \quad R = \frac{16a^* \nu^2 \sigma T_\infty^3}{k u_0^2} \\ M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \quad K' = \frac{\nu^2 K}{u_0^2}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D}, \quad K_l = \frac{\nu K_l'}{u_0^2}, \quad Q = \frac{u_0^2 S}{\nu} \end{aligned} \right\} \dots (8)$$

In equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr \theta + Gc \phi + \frac{\partial^2 U}{\partial Y^2} - \left(M + \frac{1}{K} \right) U \quad \dots (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr} \theta + S \theta \quad \dots (10)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial Y^2} - K_l \phi \quad \dots (11)$$

The initial and boundary conditions in dimensionless form are as follows:

$$\left. \begin{aligned} t \leq 0: U = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{for all } Y \\ t > 0: U = 1, \quad \theta = t, \quad \phi = t \quad \text{at } Y = 0 \\ U \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } Y \rightarrow \infty \end{aligned} \right\} \dots (12)$$

All the physical variables are defined in the nomenclature the solutions are obtained for hydro magnetic flow field in the presence of thermal radiation, heat absorption and chemical reaction of first order.

The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual finite difference technique and the solutions are derived as follows:

3. Method of Solution

The governing Equations (9), (10) and (11) are to be solved under the initial and boundary conditions of equation (12). The finite difference method is applied to solve these equations.

The equivalent finite difference scheme of equations (9), (10) and (11) are given by

$$\left[\frac{U_{i,j+1} - U_{i,j}}{\Delta t} \right] = Gr\theta_{i,j} + Gc\phi_{i,j} + \left[\frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{(\Delta y)^2} \right] - \left[M + \frac{1}{K} \right] U_{i,j} \dots (13)$$

$$\left[\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} \right] = \frac{1}{Pr} \left[\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2} \right] - \frac{R}{Pr} \theta_{i,j} + S\theta_{i,j} \dots (14)$$

$$\left[\frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta t} \right] = \frac{1}{Pr} \left[\frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{(\Delta y)^2} \right] - K_1\phi_{i,j} \dots (15)$$

Here, index i refers to Y and j to time. The mesh system is divided by taking, $\Delta y = 0.1$.

From the initial conditions in Equation (12), we have the following equivalent.

$$\left. \begin{aligned} u(0,0) = 0, \quad \theta(0,0) = 0, \quad \phi(0,0) = 0 \\ u(i,0) = 0, \quad \theta(i,0) = 0 \quad \text{for all } i \text{ except } i = 0 \end{aligned} \right\} \dots (7)$$

The boundary conditions from equation (7) are expressed in finite difference form are as follows:

$$\left. \begin{aligned} u(0,j) = 1, \quad \theta(0,j) = t, \quad \phi(0,j) = t \quad \text{for all } j \\ u(1,j) = 0, \quad \theta(1,j) = 0, \quad \phi(1,j) = 0 \quad \text{for all } j \end{aligned} \right\} \dots (8)$$

Here, infinity is taken as $y = 6$. First, the velocity profile at the end of time step namely $u(i, j + 1)$, $i = 1$ to 10 is computed from equation

(13), the temperature profile $\theta(i, j + 1)$, $i = 1$ to 10 from equation (14) and wall concentration

profile $\phi(i, j + 1)$, $i = 1$ to 10 from equation (15). The procedure is repeated until $t = 1$ (i.e., $j = 800$). During computation, Δt was chosen to be 0.00125. These computations are carried out for different values of parameters Gr, Gc, Pr, Sc, S (Heat source

parameter), Magnetic field parameter (M), Porosity parameter (K), radiation Parameter (R), Chemical reaction parameter (K_1) and t (time). To judge the accuracy of the convergence of the finite difference scheme, the same program was run with smaller values of Δt , i.e., $\Delta t = 0.0009, 0.001$ and no significant change was observed. Hence, we conclude that the finite difference scheme is stable and convergent.

4. Results and Discussion

The numerical values of the velocity, temperature and wall concentration profiles are computed for different parameters like as Magnetic field parameter (M), Porosity parameter (K), Heat source parameter (S), Radiation Parameter (R), Chemical reaction parameter (K_1), Grashoff number (Gr), Modified Grashoff number (Gc), Prandtl number (Pr), Schmidt number (Sc) and time (t). The purpose of the calculations given here is to study the effects of the given various parameters upon the nature of the flow and transport.

Figures-(1) to (9) illustrates the effect of the velocity profile of fluid for different parameters at time $t = 0.2$. Separately it is found that the velocity decreases continuously with increases in Y . Figure – (1) shows the variation of velocity U with magnetic parameter (M). It is observed that the velocity decreases as M increases. Figure – (2) shows that an increase in porosity parameter K causes an increase in velocity profile of fluid. From Figure – (3), it is observed that the velocity of fluid increases as the Grashoff number Gr increase. The variation of U with

modified Grashoff number Gc is shown in Figure – (4). It is noticed that increase in Gc leads to increase in velocity of fluid. From Figure – (5) shows the variation of velocity U with Prandtl number Pr . It is observed that the velocity of fluid decreases as Pr increases. The variation of U with thermal radiation R is shown in Figure – (6). It is noticed that increase in R leads to decrease in velocity of fluid. Figure – (7), shows the variation of velocity profile of fluid U with heat source parameter S . It is observed that the velocity increases as S increases. The velocity profile of fluid for Schmidt number Sc is shown in figure – (8). It is clear that velocity of dusty fluid u decreases with increasing in Sc . In figure – (9), the velocity profile U of fluid decreases due to increasing Chemical reaction parameter K_1 .

Figures-(10) to (12) demonstrates the effect of the temperature profile of fluid for different parameters at time $t = 0.2$. From Figure – (10), the variation of θ with thermal radiation R is shown in Figure – (11). It is noticed that increase in R leads to decrease in temperature profile of fluid. It is observed that increase in Prandtl number Pr causes decrease in temperature profile θ of fluid. Figure – (12) shows that an increase in heat source parameter S causes an increase in temperature profile θ of fluid.

Figures-(13) to (14) represents the effect of the concentration profile of fluid for different parameters at time $t = 0.2$. From Figure – (13), it is noticed that an increase in Schmidt number Sc leads to decrease in concentration profile of fluid. Figure – (14) shows that an increase in chemical reaction parameter K_1 causes decrease in concentration profile of fluid.

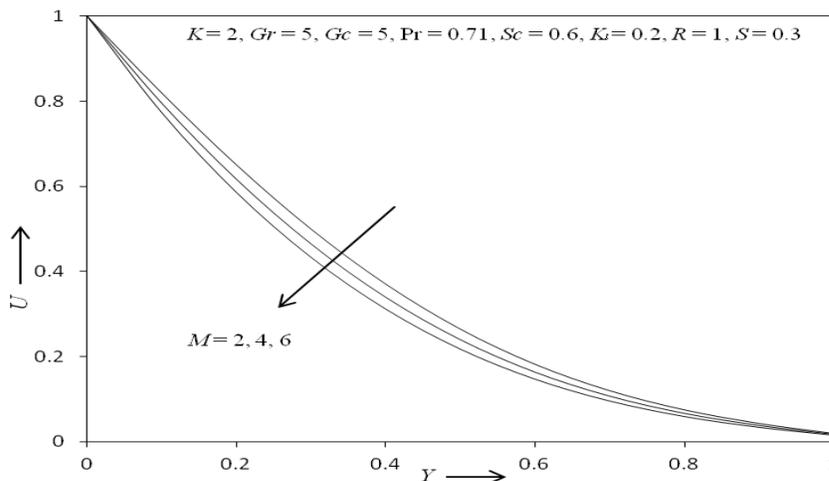


Fig. - 1: Velocity profile for the different value of M ,

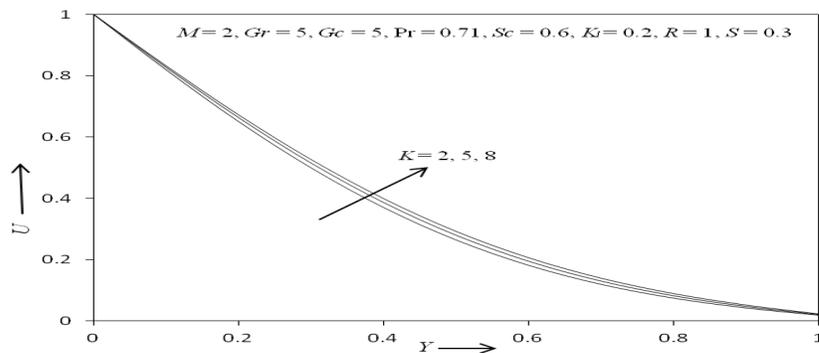


Fig. - 2: Velocity profile for the different value of K .

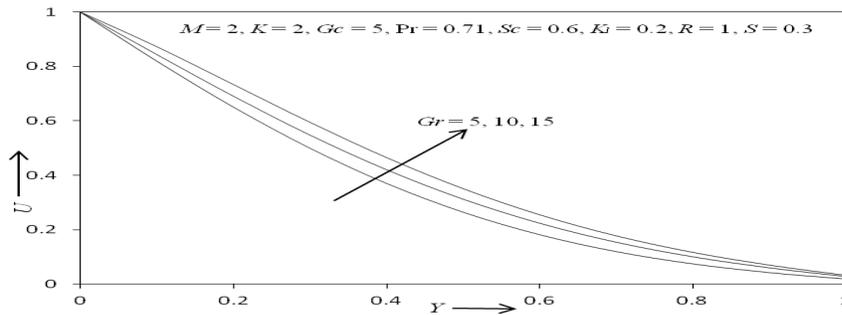


Fig. - 3: Velocity profile for the different value of Gr .

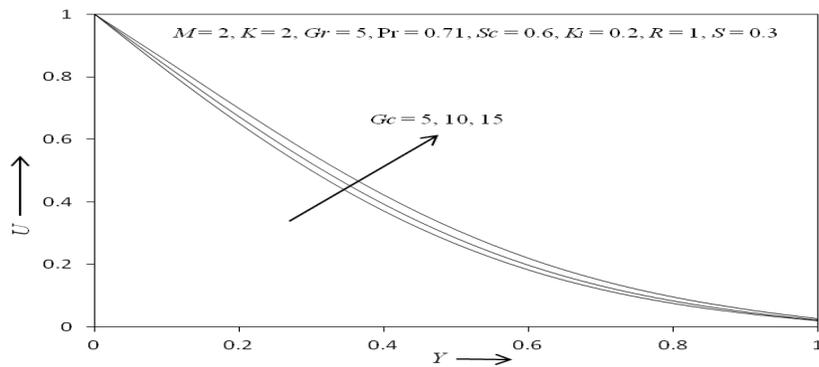


Fig. - 4: Velocity profile for the different value of G_c .

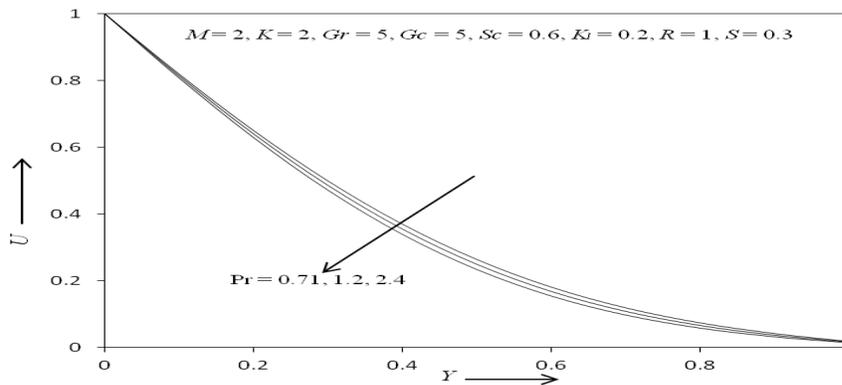


Fig. - 5: Velocity profile for the different value of Pr .

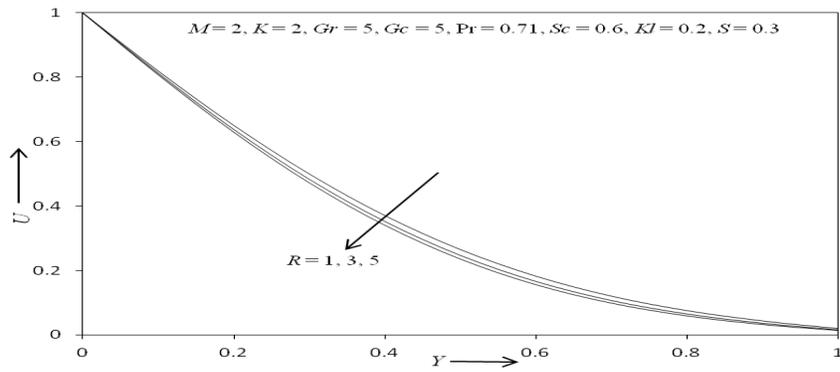


Fig. - 6: Velocity profile for the different value of R .

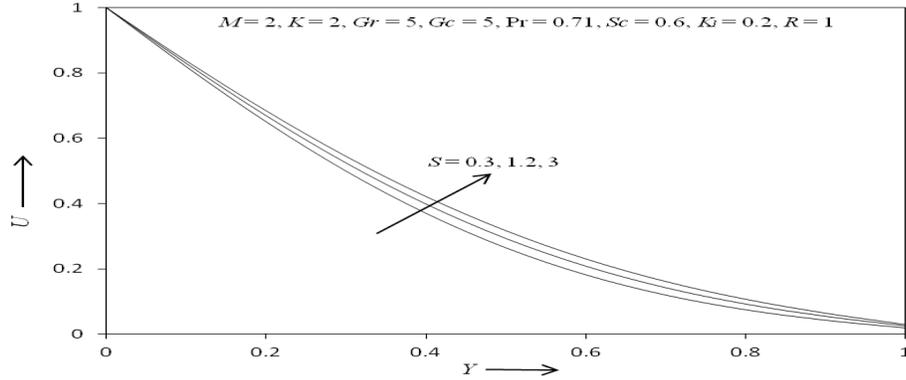


Fig. - 7: Velocity profile for the different value of S .

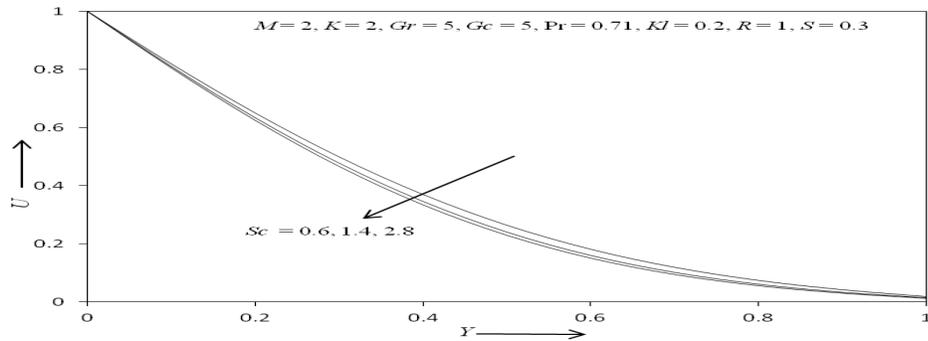


Fig. - 8: Velocity profile for the different value of Sc .

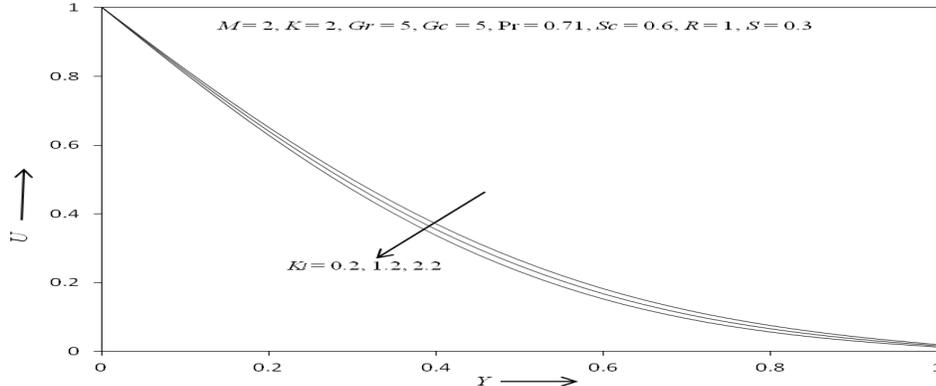


Fig. - 9: Velocity profile for the different value of Kl .

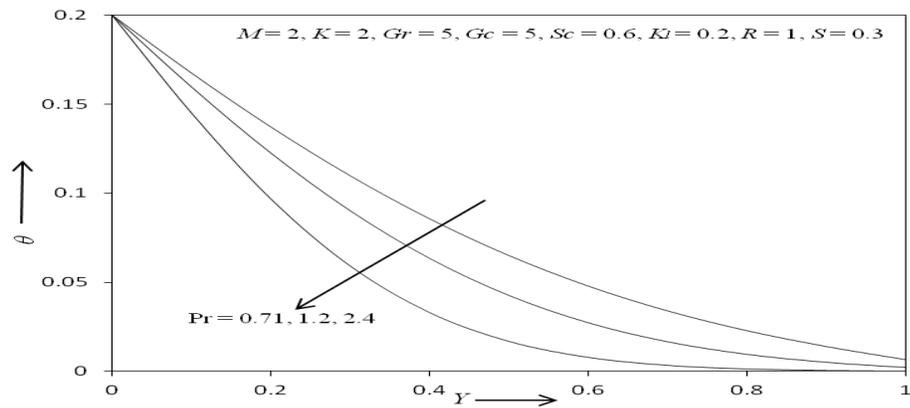


Fig. - 10: Temperature profile for the different value of Pr.

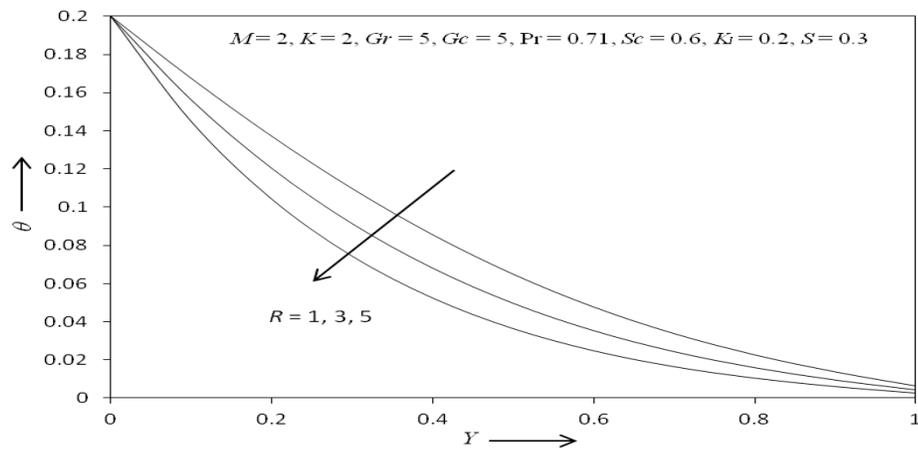


Fig. - 11: Temperature profile for the different value of R.

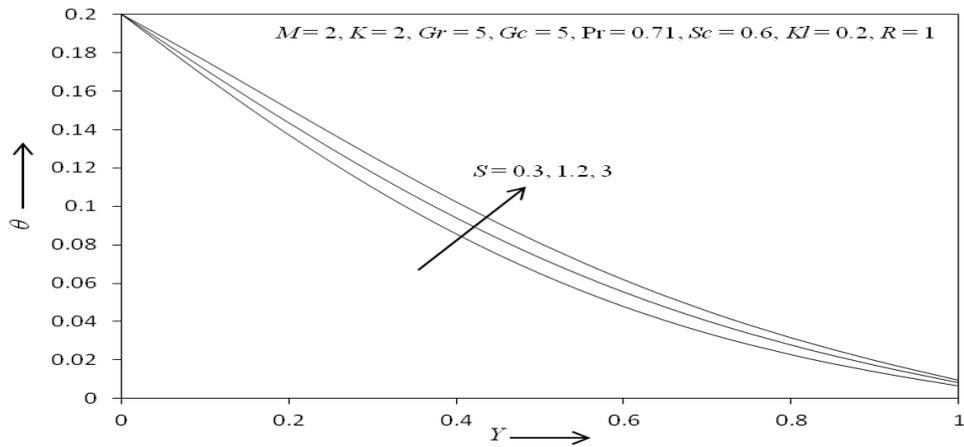
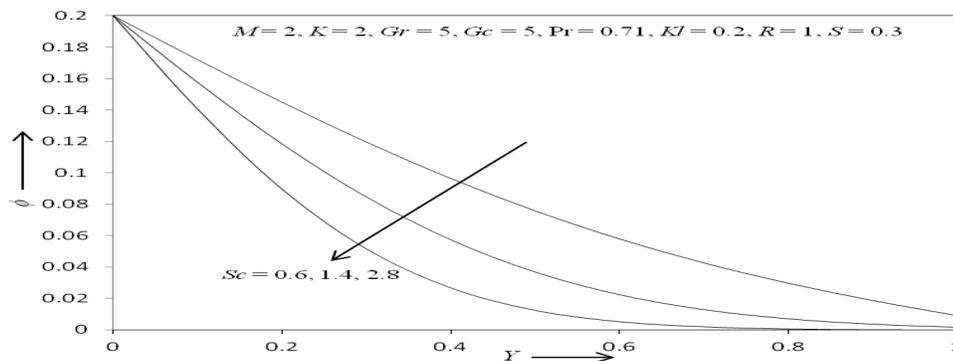
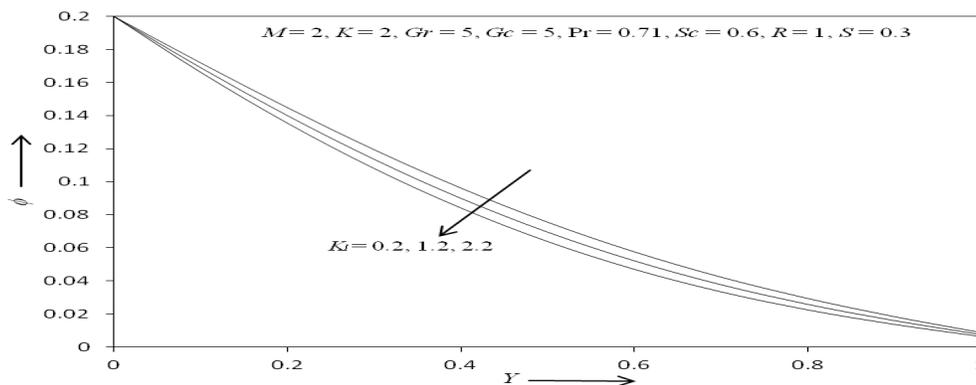


Fig. - 12: Temperature profile for the different value of S.

Fig. - 13: Concentration profile for the different value of Sc .Fig. - 14: Concentration profile for the different value of KI .

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