Economic Load Dispatch Problem and Mat lab Programming of Different Methods

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Abstract

Economic Load Dispatch (ELD) is one of an important optimization tasks and operational decision which provides an economic condition for power systems. This paper presents overview of economic load dispatch problems and solution methodologies. MATLAB programming of different methods used for solving economic load dispatch problem was done and results are given in tabular form. Lambda iteration method is implemented for three cases of

- Without generation limits and losses
- With generation limits and without losses and
- With generation limits and losses. Newton's and the dynamic programming methods were put into action for coding.

1. Introduction

The efficient and optimum economic operation of electric power systems has always occupied an important position in electric power industry. In recent decades, it is becoming very important for utilities to run their power systems with minimum cost while satisfying their customer demand all the time and trying to make profit. With limited availability of generating units and the large increase in power demand, fuel cost and supply limitation, the committed units should serve the expected load demand with the changes in fuel cost and the uncertainties in the load demand forecast in all the different time intervals in an optimal manner[1],[2],[3]&[4].

The basic objective of ELD of electric power generation is to schedule the committed generating unit outputs, so as to meet the load demand at minimum operating cost while satisfying all unit and system equality and inequality constraints. The ELD problem has been tackled by many researchers in the past. ELD problem involves different problems. The first is Unit Commitment or pre-dispatch problem where in it is required to select optimally out of the available generating sources to operate to meet the expected load and provide a specified margin of operating reserve over a specified period of time. The second aspect of ELD is on-line economic dispatch where in it is required to distribute the load among the generating units actually parallel with the system in such a manner as to minimize the total cost of supplying power. In case of ELD, The generations are not fixed but they are allowed to take values again within certain limits so as to meet a particular load demand with minimum fuel consumption [5], [6].

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2. Problem Formulation

The ELD problem is defined as to minimize the total operating cost of a power system while meeting the total load plus transmission losses within generator limits. Mathematically the problem is defined as (including losses) by equation (1) given as [1]

Minimize:

$$F_i(P_i) = \sum_{i=1}^{n_g} (\gamma_i P_i^2 + \beta_i P_i + \alpha_i) \quad (1)$$

Subject to (1) the energy balance equation

$$\sum_{i=1}^{n_g} P_i = P_D + P_L \quad (2)$$

(2) the inequality constraints

$$P_{i(min)} \le P_i \le P_{i(max)} \quad (3)$$

Where α_i, β_i, γ_i are cost coefficients , P_D is load demand, P_i is real power generation, P_L is power transmission loss and n_g is number of generation busses.

One of the most important, simple but approximate method of expressing transmission loss as function of generator powers is through B-coefficients. This method uses the fact that under normal operating condition, the transmission loss is quadratic in the injected bus real power. The general form of the loss formula using B-coefficient is given by eqn(4)

$$P_L = \sum_{i=1}^{n_g}\sum_{j=1}^{n_g} P_i B_{ij} P_j \quad \text{MW} \qquad (4)$$

Where P_i, P_j are real power injection at the i^{th}, j^{th} buses and B_{ij} are loss coefficients which are constant under certain assumed conditions [8], [9], & [11]. The above loss formula is known as the George's formula. Another more accurate form of transmission loss expression, frequently known as the Kron'lossf ormula is eqn (5)

$$P_L = \sum_{i=1}^{n_g}\sum_{j=1}^{n_g} P_i B_{ij} P_j + \sum_{i=1}^{n_g} B_{0i} P_i + B_{00} \quad \text{MW} \qquad (5)$$

where, B_{00}, B_{i0}, and B_{ij} are the loss coefficient which are constant under certain assumed conditions. The above constrained optimization problem is converted into an unconstrained optimization problem. Lagrange multiplier method is used in which a function minimized (or maximized) is subjected to side conditions in the form of equality constraints. Using Lagrange multipliers, an augmented function is defined by eqn (6)

$$L(P_i, \lambda) = F(P_i) + \lambda(P_D - P_L - \sum_{i=1}^{n_g} P_i) \qquad (6)$$

Where, λ is the Lagrangian multiplier. Necessary conditions for the optimization problem are

$$\frac{\partial L(P_i, \lambda)}{\partial P_i} = \frac{\partial F(P_i)}{\partial P_i} + \lambda\left(\frac{\partial P_L}{\partial P_i} - 1\right) = 0, \ (i = 1, 2 \ldots n_g) \qquad (7)$$

Rearranging the above equation

$$\frac{\partial F(P_i)}{\partial P} = \lambda_i\left(1 - \frac{\partial P_L}{\partial P_i}\right., \ (i=1, 2, 3 \ldots n_g) \qquad (8)$$

Where, $\frac{\partial F(P_i)}{\partial P_i}$: Incremental cost of the i^{th} generator (Rs/MW h) Equation (9) is known as the exact coordination equation and

$$\frac{\partial L(P_i, \lambda)}{\partial \lambda} = P_D + P_L - \sum_{i=1}^{n_g} P_i = 0 \qquad (9)$$

By differentiating the transmission loss eqn (5) with respect to P_i, the incremental transmission loss can be obtained as

$$\frac{\partial P_L}{\partial P_i} = \sum_{j=1}^{n_g} 2 B_{ij} P_j \qquad (10)$$

and by differentiating the cost function of Eqn (1) with respect to P_i , the incremental cost can be obtained as

$$\frac{\partial F(P_i)}{\partial P_i} = 2\gamma_i P_i + \beta_i, \ (i=1, 2, 3 \ldots\ldots\ldots n_g) \qquad (11)$$

to find the solution $2\gamma_i P_i + \beta_i = \lambda(1 - 2\sum_{i=1}^{n_g} B_{ij} P_j)$

$(i=1, 2, 3 \ldots\ldots\ldots n_g)$ (12)

Rearranging the eqn(12) to get P_i, i.e.

$$2\gamma_i P_i + \beta_i = \lambda(1 - 2b_{ij} P_i - \sum_{i=1}^{n_g} 2 B_{ij} P_j)$$

$(i=1, 2, 3 \ldots\ldots\ldots n_g)$

$$2(\gamma_i + \lambda b_{ij}) P_i + 2\sum_{j=1}^{n_g} B_{ij} P_j = (\lambda - \beta_i)$$

$(i=1, 2, 3 \ldots\ldots\ldots n_g)$ (13)

The above linear eqn(13) can be solved to obtain the value of P_i if λ is known.

3. Lambda Iteration Method

The solution to this problem can be approached by considering a graphical technique for solving the problem and then extending this into the area of computer algorithms. The lambda-iteration procedure converges very rapidly for this particular type of optimization problem. The actual computational procedure is slightly more complex [1], [12].

We use following MATLAB code formulated for no losses and no generation losses as:

```
for i=1:3
n(i)=(f(i,2)/(2*f(i,3)));
p(i)=(1/(2*f(i,3)));
end
m=sum(n);
q=sum(p);
lambda = ((Demand+m)/q);
disp(lambda)
for i=1:3
P(i)=((lambda-f(i,2))/(2*f(i,3)));
end
P
for i=1:3
   totalcost=(f(i,1)+(f(i,2)*P)+(f(i,3)*P*P));
end
```

For inclusion of losses:

```
Ploss = 0.00003*P
(1,1)^2+0.00009*P(1,2)^2+0.00012*P(1,3)^2
for j=1:4
for i=1:3
n(i)=f(i,2)/(2*f(i,3));
w(i)=(1-(2*PL(1,i)*P(1,i)))/(2*f(i,3));
end
m=sum(n);
q=sum(w);
lambda=(PD+Ploss+m)/q
for i=1:3
P(i)=(lambda*(1-2*PL(1,i)*P(1,i))-f(i,2))/(2*f(i,3));
end
j=j+1;
end
    disp(P)
```

4. Newton's Method

The economic dispatch can also be solved by observing that the aim is to always drive

$\nabla L_x = 0$ (14)

Since this is a vector function, the problem can be formulated as one of finding the correction that exactly drives the gradient to zero (i.e., to a vector, all of whose elements are zero). Newton's method can be used to find this. Newton's method for a function of more than one variable is developed as follows [1], [13]&[14].

Suppose for the function **g(x)** to be driven to zero. The function **g** is a vector and the unknowns, **x,** are also vectors. Then, to use Newton's method eqn(15):

$g(x+\Delta x) = g(x) + [g'(x)]\Delta x = 0$ (15)

If the function is defined as:

$$g(x) = \begin{bmatrix} g_1(x_1, x_2, x_3) \\ g_2(x_1, x_2, x_3) \\ g_3(x_1, x_2, x_3) \end{bmatrix}$$

then

$$g'(x) = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \frac{\partial g_1}{\partial x_3} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \frac{\partial g_2}{\partial x_3} \\ \frac{\partial g_3}{\partial x_1} & \frac{\partial g_3}{\partial x_2} & \frac{\partial g_3}{\partial x_3} \end{bmatrix}$$

Which is the familiar Jacobian matrix. The adjustment at each step is then $\Delta x = -[g'(x)]^{-1} g(x)$

Now, if the g function is the gradient vector ∇L_x, then $\Delta x = -inv\left[\frac{\partial}{\partial x}\nabla L_x\right] . \Delta L$

For economic dispatch problem this takes the form eqn(16):

$L = \sum_{i=1}^{N} F_i(P_i) + \lambda(P_{load} - \sum_{i=1}^{N} P_i)$ (16)

and ∇L is as it was defined before. The Jacobian matrix now becomes one made up of second derivatives and is called the Hessian matrix:

$$\left[\frac{\partial}{\partial x}\nabla L_x\right] = \begin{bmatrix} \frac{d^2L}{dx_1^2} & \frac{d^2L}{dx_1 dx_2} & \cdots \\ \frac{d^2L}{dx_2 dx_1} & \cdots & \\ \vdots & \vdots & \\ \frac{d^2L}{d\lambda dx_1} & \cdots & \end{bmatrix}$$

Generally, Newton's method will solve for the correction that is much closer to the minimum generation cost in one step than would the gradient method.

We use following MATLAB code formulated as:

```
Alpha=X;
Beta =Y;
Gamma =Z;
Lambda=input ('Enter the value of lambda :');
P_load=A
P_sum=sum(P_initial(1)+P_initial(2)+P_initial(3))
%......Formation of Hessian Matrix......
for i=1:3
   L(i)=beta(i)+2*gamma(i)*P_initial(i);
end
dellambda=[L(1)-Lambda;L(2)-Lambda;L(3)-
Lambda;P_load-P_sum]
G=2*gamma;
H= [G(1) 0 0 -1;0 G(2) 0 -1;0 0 G(3) -1;-1 -1 -1 0]
M= inv(H)
delp_Power=-(M*dellambda)
Lambda_optimal=delp_Power(4)

P_optimal=(P_initial')+delp_Power;
```

5. Dynamic Programming Method

The application of digital methods to solve a wide variety of control and dynamics optimization problems in the late 1950s led Dr. Richard Bellman and his associates to the development of dynamic programming. These techniques are useful in solving a variety of problems and can greatly reduce the computational effort in finding optimal trajectories or control policies. The theoretical mathematical background, based on the calculus of variations, is somewhat difficult. The applications are not, however, since they depend on a willingness to express the particular optimization problem in terms appropriate for a dynamic-programming (DP) formulation [1].

In the scheduling of power generation systems, DP techniques have been developed for the economic dispatch of thermal systems, the solution of hydrothermal economic-scheduling problems and the practical solution of the unit commitment problem. If valve points are considered in the input-output curve, the possibility of non-convex curves must be accounted for if extreme accuracy is desired. If non-convex input-output curves are to be used, an equal incremental cost methodology cannot be used since there are multiple values of MW output for any given value of incremental cost.

Under such circumstances, there is a way to find an optimum dispatch which uses dynamic programming (DP). The dynamic programming solution to economic dispatch is done as an allocation problem. Using this approach, not just a single optimum set of generator MW outputs is calculated for a specific total load supplied-rather a set of outputs are generated, at discrete points, for an entire set of load values[16]. One problem that is common to economic dispatch with dynamic programming is the poor control performance of the generators. When the economic dispatch is to be done with dynamic programming and the cost curves are non-convex, a difficult problem is encountered whenever a small increment in load results in a new dispatch that calls for one or more generators to drop their output a great deal and others to increase a large amount. The resulting dispatch may be at the most economic values as determined by the DP, but the control action is not acceptable and will probably violate the ramp rates for several of the units.

The only way to produce a dispatch that is acceptable to the control system, as well as being the optimum economically, is to add the ramp rate limits to the economic dispatch formulation itself. This requires a short-range load forecast to determine the most likely load and load-ramping requirements of the units. This problem can be stated as follows [17], [18].

Given a load to be supplied at time increments *t* = 1 . . . t_{max}, with load levels of P_{load}^{t} and N generators on-line to supply the load:

$$\sum_{i=1}^{N} P_i^t = P_{load}^t \quad (17)$$

Each unit must obey a rate limit such that:

$P_i^{t+1} = P_i^t + \Delta P_i$

And $-\Delta P_i^{max} \leq \Delta P_i \leq \Delta P_i^{max}$

Then the units must be scheduled to minimize the cost to deliver power over the time period as:

$$F^{total} = \sum_{t=1}^{T_{max}} \sum_{i=1}^{N} F_i(P_i^t) \quad (18)$$

Subject to: $\sum_{i=1}^{N} P_i^t = P_{load}^t \quad for\ t = 1 \ldots\ldots t_{max}$ (19)

And $P_i^{t+1} = P_i^t + \Delta P_i$

With $-\Delta P_i^{max} \leq \Delta P_i \leq \Delta P_i^{max}$

This optimization problem can be solved with dynamic programming and the "control performance" of the dispatch will be considerably better than that using dynamic programming and no ramp limit constraints.

6. Result And Discussions

Lambda iteration method is implemented on 3 cases of without losses and generation limits, with losses and with generation limits. Newton's method was used and the dynamic programming method for 1 m/c, two m/c and 3 m/c was used. The programs were written on MATLAB 9 and implemented on Intel Core 2 Duo having 2.4 GHZ 3GB RAM.

6.1 Lambda Iteration method

MATLAB promming of three kind of economic dispatch problem was done namely without losses and generation limits, with generation limits and with losses. Problem taken from Wood and Wollenberg [1], p 36 given as,

The heat rates for three thermal plants in MBtu/h are given as: $H_1 = 510.0 + 7.2\ P_1 + 0.00142\ P_1^2$, $H_2 = 310.0 + 7.85P_2 + 0.00194\ P_2^2$, $H_3 = 78.0 + 7.97\ P_3 + 0.00482\ P_3^2$. With the following fuel costs: Unit 1 = 1.1 Rs/MBtu, Unit 2 = 1.0 Rs/MBtu, Unit 3 = 1.0 Rs/MBtu and total load of 850MW.Results obtained are summarized in TABLEI.

In TABLE II, lambda iteration method is applied on economic load dispatch problem with generation limits whose problem considered is given by-

The fuel cost functions for 3 thermal plants in Rs/hr are, $C_1 = 500 + 5.3\ P_1 + 0.004\ P_1^2$, $C_2 = 400 + 5.5\ P_2 + 0.006\ P_2^2$, $C_3 = 200 + 5.8\ P_3 + 0.009\ P_3^2$, the total load P_D, is 975 MW with the following generator limits (in MW): $200 \leq P_1 \leq 450$, $150 \leq P_2 \leq 350$, $100 \leq P_3 \leq 225$.

In TABLE III, lambda iteration method is applied on economic load dispatch problem with losses considered whose problem considered is same as problem one with the loss expression given by $P_L = 0.00003\ P_1^2 + 0.00009\ P_2^2 + 0.00012\ P_3^2$.

$\lambda_{initial}$	$P_{total\_initial}$	delP	dellambda	$\lambda_{optimal}$	$P1_{optimal}$	$P2_{optimal}$	$P3_{optimal}$	Total Cost
2	-915.2778	1890.3	7.1632	9.4000	450	325	200	8236.3
7	404.1667	570.8333	2.1632	9.4000	450	325	200	8236.3
8	668.0556	306.944	1.1632	9.400	450	325	200	8233

Table: 1. Economic Load Dispatch result for Lambda Iteration Method

Parameter	Value
P_{loss}	15.600
Lambda	9.5279
$P1_{optimal}$	435.0215
$P2_{optimal}$	299.8997
$P3_{optimal}$	130.6788

P_{loss}	15.8211

Table: 2. Economic Load Dispatch result for Lambda Iteration Method

6.2 Newton's Method

MATLAB priming of Newton's method was done. Problem was taken as, The fuel cost functions for 3 thermal plants in Rs/hr are $C_1 = 561 + 7.92\ P_1 + 0.001562\ P_1^2$, $C_2 = 310 + 7.85\ P_2 + 0.001940 P_2^2$, $C_3 = 78 + 7.97\ P_3 + 0.00482\ P_3^2$ with total load varied.

$\lambda_{initial}$	P_{load}	$P1_{initial}$	$P2_{initial}$	$P3_{initial}$	P_{sum}	$\lambda_{optimal}$	$P1_{optimal}$	$P2_{optimal}$	$P3_{optimal}$
1	800	450	325	200	975	9.0749	369.68	315.69	114.61
0	800	300	200	300	800	9.0749	369.68	315.69	114.61
8	600	250	100	150	500	8.78	275.75	140.08	84.17
4	900	400	300	200	900	9.22	416.65	353.51	129.83
8	900	300	250	350	900	9.22	416.65	353.51	129.83

Table: 3. Economic Load Dispatch result for Newton Method

6.3 Dynamic Programming Method

MATLAB priming of dynamic programming method was done. Problem taken from Wood and Wollenberg [1], p 51 given as, There are three units in the system; all are on-line. Their input-output characteristics are neither smooth nor convex. Data are as follows.

Power Levels (MW) $P_1 = P_2 = P_3$	Costs (Rs/hour) F_1	Costs (Rs/hour) F_2	Costs (Rs/hour) F_3
0	∞	∞	∞
50	810	750	806
75	1355	1155	1108.5
100	1460	1360	1411
125	1772.5	1655	11704.5
150	2085	1950	1998
175	2427.5	∞	2358
200	2760	∞	∞
225	∞	∞	∞

In this problem, 10000 have been taken in place of infinity

Demand	Cost	P_1	P_2	P_3
300	4168	150	100	50
250	3558	150	50	50
200	2971	100	50	50
250	3558	150	50	50

Table: 4. Economic Load Dispatch result for Dynamic Programming Method

7. Conclusion

This paper gives overview of economic load dispatch problems and solution methodologies. Implementation is done using MATLAB programming and results are given in tabular form. Conventional method like lambda iteration method converges rapidly but complexities increases as system size increase also lambda method always requires that one be able to find the power output of a generator, given an incremental cost for that generator. In cases where cost function is much more complex, method like gradient and Newton can be used. If non convex input-output curves are to be used dynamic programming can be used to solve economic dispatch problems. Hence different methods have different applications.

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