

# Distribution free Variable Step Size Normalized Least Means Square Algorithm

Demian Kosale

Department of Electrical Engineering, Vishwavidyalaya Engineering College, Lakhanpur, Ambikapur (C.G.), India

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## Abstract

In this paper, we present a new approach to improve the acoustic echoes occurs in today's telecommunication system. We show that the new approach reduces the computational complexity; unlike old approaches like a LMS, NLMS and variable step size (VSS) algorithm, converges rate and misadjustment and computational complexity are biggest challenges for us.

So new approach is known as DFVSS-NLMS algorithm and suitable candidate for a real world application.

## 1. Introduction

Acoustic echo cancellation is one of the most popular applications of adaptive filter [1]. The role of the adaptive filter is to identify the acoustic echo path between the terminals loudspeaker and microphone.

Even though many interesting adaptive filtering algorithm have been developed and are applicable for acoustic echo cancellation [2], an application with limited precision and processing power, the least means-square (NLMS) algorithm [3] (e.g., frequency domain or subband versions [1]) are usually applied.

The standard least means square (LMS) algorithm is considered to be one of the simplest algorithms for adaptive filtering, but it is sensitive to the scaling of its input when choosing a step-size parameter to guarantee stability [2],[3].

The NLMS algorithms solve this problem by normalizing with the power of the input. For both algorithms, the parameter of step-size governs the convergence speed and the steady-state excess mean-square error. To better tradeoff the conflicting requirement of fast convergence rate and low misadjustment, various schemes for adjusting the step-size have been reported [4], [5], [6], [7]. To meet these conflicting requirements, the step size needs to be controlled. Thus, a number of variable step size NLMS (VSS-NLMS) algorithms have been proposed [8], [9] and references therein. In [5], elaborated and distribution free VSS-NLMS (DFVSS-NLMS) is proposed. This algorithm is gives the good performance in the context of acoustic echo cancellation [AEC].

## 2. Basic Concepts of Echo Cancellation

In acoustic echo cancelation, the estimates of the near-end echo path response is computed which is used to generate an estimate of echo. The estimate of echo is subtracted from the near-end microphone output to subtract the actual echo.

### Corresponding Author,

E-mail address: dkosale@gmail.com

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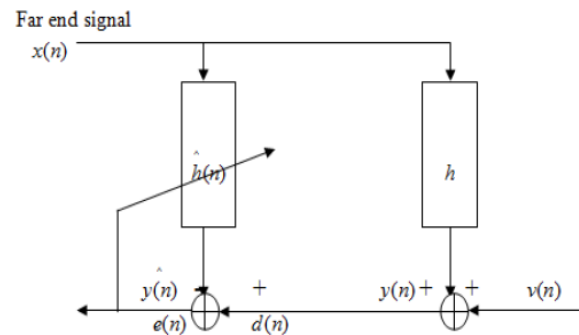


Fig. 1. Block diagram of the echo canceller

Where,

$x(n)$  Far-end signal

$v(n)$  Near-end signal

$d(n)$  Echo or desired signal

The problem then reduces to similar to the room echo

path response  $h$  by an impulse response  $\hat{h}(n)$  of the adaptive filter. So that feeding a same input to the adaptive

filter the estimate of actual echo,  $\hat{y}(n)$  is obtained. The use of adaptive filter in the echo cancellation is necessary because the path of echo's are highly time varying, so that the use of fixed filter is not suitable.

## 3. Basics Problems

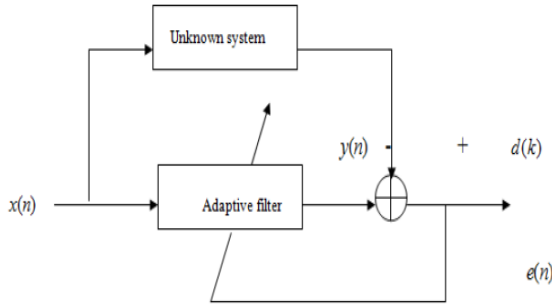
In hand free telephony, the objective is to permit two or more people, sitting in two different rooms, two converge with each other. In simple configuration, there are two separate rooms one is far end room and another is near end room. Each room contains a microphone and a loudspeaker pair which is used by one speaker to converge with other..

The far-end signal broadcast to the near end signal  $x(n)$  is broadcast to the near end room. The near end room has a microphone which is for the use of near end speaker but this near end speaker also receives a delayed and distorted

version of the far end signal  $x(n)$  as an echo  $d(n)$  due to the room.

**3.1 System Identification**

System identification refers to the ability of an adaptive system to find the FIR filter that best reproduces the response of another system, whose frequency response is a priori unknown. System identification is mostly used in divergence application, setup is given below Fig2.



**Fig: 2.** System Identification

The FIR filter reproduces the behavior of the 'unknown system'. This works perfectly when the system to be identified has got a frequency response that matches with that of a certain FIR filter.

But if the unknown system is an all-pole filter, then the FIR filter will try its best. It will never be able to give zero output but it may reduce it by converging to an optimum weights vector. The frequency response of the FIR filter will not be exactly equal to that of the 'unknown system' but it will certainly be the best approximation to it.

Let us consider that the unknown filter is a time invariant, which indicate that the coefficient of the impulse response are constant and of finite extent (FIR). Therefore,

$$d(n) = \sum_{k=0}^{N-1} h_k x(n-k)$$

The output of the adaptive filter with the same number of the coefficient N, is given by,

$$y(n) = \sum_{k=0}^{N-1} w_k x(n-k)$$

These two systems to be equal, the difference between  $e(n) = d(n) - y(n)$  must be equal to zero. Under these conditions, the two set of the coefficients are also equal. It is the method of adaptive filtering that will enable us to produce an error,  $e(n)$  approximately equal to zero and therefore will identify that.  $w_k \approx h_k$ .

**4. Distribution Free VSS-NLMS Algorithm**

The DFVSS adaptive algorithm that uses the power estimate of the background noise in order to control its step-size parameter and the step size of the proposed algorithm is adjusted according to the square of a time-averaging estimate of the autocorrelation of a priori and a posteriori error.

This section described the model for a proposed algorithm.

The output of the system is given by

$$y(n) = h^T x(n) + v(n) \tag{1}$$

Where the  $n$  is the time index and  $T$  denote the transposition,

$$h = [h_0 \quad h_1 \quad \dots \quad h_{L-1}]^T \tag{2}$$

Where  $h$  is the impulse response of the unknown system of length  $L$ , that try to identify with an adaptive filter,

$$x(n) = [x(n) \quad x(n-1) \quad \dots \quad x(n-L+1)] \tag{3}$$

$x(n)$  is a vector containing a most recent sample of the system input signal and  $x(n)$  and  $v(n)$  is the system noise and indepenet with each other and also assumed that the  $x(n)$  is stationary.

**5. Derivation of Algorithm**

We defined a priori and posteriori error signal,

$$\begin{aligned} e(n) &= y(n) - \hat{h}^T(n-1)x(n) \\ &= x^T(n) [h - \hat{h}(n-1)] + v(n) \end{aligned}$$

$$\begin{aligned} \varepsilon(n) &= y(n) - \hat{h}^T(n)x(n) \\ &= x^T(n) [h - \hat{h}(n)] + v(n) \end{aligned} \tag{4}$$

Where  $L$  is length of filter,  $\hat{h}(n-1)$  and  $\hat{h}(n)$  are the estimates of the system  $h$  time  $n-1$  and  $n$ . Linear weight updated equation are written as follow:

$$\hat{h}(n) = \hat{h}(n-1) + \mu x(n)e(n)$$

Where  $\mu(n)$  is known as step size. One way to derive a  $\mu(n)$  that makes (4) to stable is to cancel the posteriori error signal.

If  $\varepsilon(n) = 0$ , but assuming that  $e(n) \neq 0, \forall n$ .

Therefore, step size is converted into  $\mu(n)_{NLMS}$ .

$$\mu(n)_{NLMS} = \hat{h}(n-1) + \frac{x(n)}{x^T(n)x(n)} e(n)$$

Above procedure makes sense in the absence of the noise, Presence of noise is cancel. (4)

$$x^T(n)[h - \hat{h}(n)] = -v(n) \neq 0, \forall n$$

$$x^T(n)[h - \hat{h}(n)] = 0, \forall n$$

We used very important assumption for the derivation of

$$\begin{aligned} \varepsilon(n) &= v(n) \\ E\{\varepsilon^2(n)\} &= \sigma_v^2, \forall n \end{aligned} \tag{5}$$

$E\{\cdot\}$  is mathematical expectation and  $\sigma_v^2 = E\{v^2(n)\}$  is the power of system noise. Using approximation  $x^T(n)x(n) = L\sigma_x^2 = LE\{x^2(n)\}$ ,

for  $L \gg 1$ , where  $\sigma_x^2$  is power of input signal and we also known that  $\mu(n)$  is deterministic nature. From equation (4) and (31), eliminating  $\hat{h}(n-1)$ , and equating to (5),

$$E\{\varepsilon^2(n)\} = \left[1 - \mu(n)L\sigma_x^2\right]\sigma_e^2(n) = \sigma_v^2$$

where  $E\{\varepsilon^2(n)\} = E\{e^2(n)\}$  is power of error signal from (33) obtained a quadratic equation,

$$\mu^2(n) - \frac{2}{L\sigma_x^2}\mu(n) + \frac{1}{(L\sigma_x^2)^2}\left[1 - \frac{\sigma_v^2}{\sigma_e^2(n)}\right] = 0$$

The step size parameter of a proposed nonparametric VSS-NLMS algorithm is given by:

$$\mu_{NPVSS}(n) = \frac{1}{x^T(n)x(n)}\left[1 - \frac{\sigma_v}{\sigma_e(n)}\right] = \mu_{NLMS}(n)\alpha(n)$$

Where  $\alpha(n)$  is normalized step size, range is given  $0 \leq \alpha(n) \leq 1$ . The NPVSS-NLMS algorithm is,

$$\hat{h}(n) = \hat{h}(n-1) + \mu_{NPVSS}(n)x(n)e(n)$$

We conclude that if  $\sigma_e(n) > \sigma_v$ , then the  $\mu_{NPVSS}(n) \approx \mu_{NLMS}(n)$ . When the algorithm starts to converge to true value,  $\sigma_e(n) \approx \sigma_v$ , and  $\mu_{NPVSS}(n) \approx 0$ . This exactly what we desired to have good convergence and low misadjustment.

NPVSS-NLMS algorithm written in terms of misalignment,  $m(n) = m(n-1) - \mu_{NPVSS}(n)x(n)e(n)$

It is understandable that  $\sigma_e(n) \geq \sigma_v \mu_{NPVSS}(n)$ , which imply that  $\mu_{NPVSS}(n) \geq 0$ . The quantity  $\sigma_e^2(n)$  is estimated as follows:

$$\sigma_e^2(n) = \lambda \sigma_e^2(n-1) + (1-\lambda)e^2(n)$$

Where  $\lambda$  is an exponential window. This estimation could result in a lower magnitude than  $\sigma_v^2$ , which would make

$\mu_{NPVSS}(n)$  negative. To overcome this problem, when its occurs is to set  $\mu_{NPVSS}(n) = 0$ .

**Table: 1.** DP VSS-NLMS Algorithm

Initialization:	$\hat{h}(0) = 0$ $\sigma_e^2(0) = 0$
Parameters:	$\lambda = 1 - \frac{1}{KL}$ , exponential window with $K \geq 2$ $\sigma_v^2$ , noise power known or estimated $\delta$ - constant, $\sigma_x^2$ , regularization $\epsilon > 0$ , very small number to avoid division by zero
Error	$e(n) = y(n) - \hat{h}(n-1)x(n)$
Update:	$\sigma_e^2(n) = \lambda \sigma_e^2(n-1) + (1-\lambda)e^2(n)$ $\beta(n) = [\delta + x^T(n)x(n)]^{-1} \left[1 - \frac{\sigma_v}{\delta + \sigma_e(n)}\right]$ $\mu_{NLMS}(n) = \begin{cases} \beta(n) & \text{if } \sigma_e(n) \geq \sigma_v \\ 0 & \text{otherwise} \end{cases}$ $\hat{h}(n) = \hat{h}(n-1) + \mu_{NLMS}(n)x(n)e(n)$

## 6. Simulation

The input signal applied to the unknown system is either a white gaussian noise or speech signal. The output of the plant is mixed with noise such that the signal to noise ratio remain 20-dB. This signal is a desired signal for adaptive filter. The error vector obtained as the difference of desired and output vector is used to update output of adaptive filter. The initial weights of are initially set to zero. The simulation study has been carried out for NLMS, NPVSS-NLMS

### 6.1 NLMS and DFVSS-NLMS

The acoustic coupling between microphone and microphone in hand free telephones generates echoes. To remove this echo, we need to identify impulse response of unknown system. Simulation results, input signal is consider as white gaussian signal or speech signal. An independent white gaussian noise signal is added to the output of unknown system at 30-dB. We also assume that power of noise signal is known. Parameters setting for simulations

are  $\sigma_e^2(0) = 0$ ,  $\delta = 20\sigma_x^2$  and  $\lambda = 1 - \frac{1}{KL}$  and  $K = 2$  for white gaussian noise signal. The performance of algorithm measured in terms of the normalized misalignment in (dB).

$$\text{Misalignment} \left( \hat{h}(t) \right) = 20 \log \left( \frac{\| \hat{h}(t) - h \|}{\| h \|^2} \right)$$

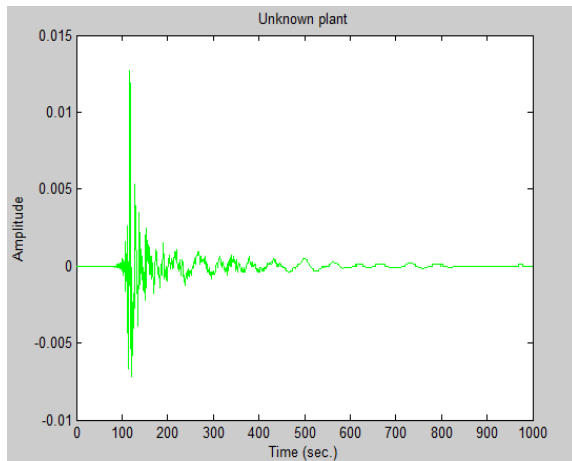


Fig. 3. Unknown plant

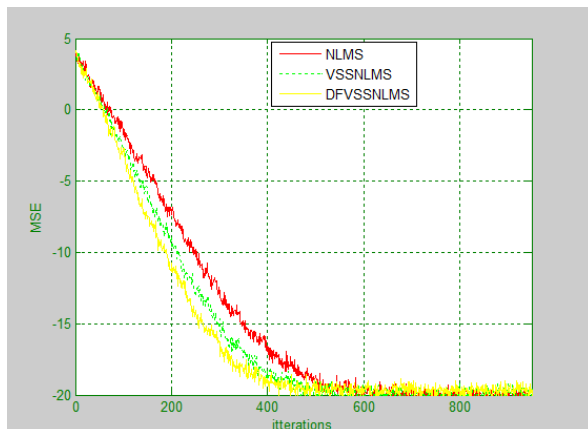


Fig. 4. Misalignment of the NLMS algorithm at

$$\left[ \delta + X^T(n)X(n) \right]^{-1}$$

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DFVSS-NLMS Algorithm. The input signal is white gaussian noise,  $L = 500$ ,  $\lambda = 1 - \frac{1}{1 - (2L)}$ , and

$$\text{SNR} = 20 \text{ dB.}$$

The simulation results show that NPVSS algorithm is better than NLMS and DFVSS-NLMS algorithm. We have compared NPVSS and NLMS and DF-VSS-NLMS. The plot has been taken between numbers of iterations and corresponding MSE. The iteration range varied from 0 to 950 where as MSE value varies from 0 dB to 5 dB. It is clear from the above plot, fig.5 that proposed algorithm converges in 20 dB signal to noise ratio, which is lesser than NLMS algorithm.

Tracking is a very important issue in adaptive algorithms. In applications like acoustic echo cancellation, it is essential that an adaptive filter tracks fast since impulse responses are not very stationary. Fig. shows that, when the impulse response has changed NLMS algorithm provides more erroneous results than the previous one, where as DFVSS-NLMS algorithm shows the same results with more efficiency compare to NLMS and NPVSS algorithm.

## 7. Conclusion

In AEC, the acoustic echo paths are extremely long. The main property of the algorithm doesn't require any priori information about acoustic environment. It can be deduced from above figures that distribution free variable step size normalized least means square adaptive algorithm perform better than the other two algorithms, NLMS and NPVSS in the context of echo cancellation. In NLMS algorithm, we need to find a compromise between fast convergence and low final misadjustment. In many applications, this compromise may not be satisfactory so a DFVSS-NLMS algorithm is required. It should be noted that the idea of proposed algorithm can be used in coincidence with other NLMS-based algorithms This improves the convergence rate and reduced the computational complexity. So it is suitable for real world application.