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Selection of Best Dental Chair for Dental Clinic using Trapezoidal Fuzzy Multiple Criteria Decision Making Model with Entropy Weights

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Abstract

In the present communication, we have implemented the concept of Intuitionistic Trapezoidal Fuzzy Numbers (ITFNs) to the study of Multiple Criteria Decision Making (MCDM) problem for evaluating the best company whose information take the form of ITFNs. We propose an algorithm for ITF-MCDM problem where the weights of the involved attributes are supposed to be completely unknown. These weights have been calculated on the basis of the decision maker's qualitative opinion to the attributes with the help of pre-defined linguistic variables and an entropy measure. Finally, the ranking of the companies has been determined by calculating the hamming distance between the ideal alternative and all the available alternatives. Selecting dental equipment is one of the most important decisions you'll make for your practice. This paper presents the facts you need to efficiently and wisely navigate your way through the equipment buying process.

1. Introduction

Making the right choice increases productivity and success. When considering dental chairs, stability is key. The chair must provide an unwavering platform for the delicate, precise work of a dental. It creates an efficient working environment that maximizes dentist's access to the patient - and provides comfort for his patient.

Following parameters are required to be considered while choosing a dental chair:

1.1 Durability and Reliability

The first thing we look for the dental equipment's durability and reliability. For an environment that's prone to moisture and constant. Equipment must be built to withstand the daily rigours of dentistry. Try these tests in a dealer showroom or at a dental tradeshow:

- Touch each piece of equipment to get a feel for how well it's made.
- Operate the dental chairs by moving the armrest up and down. Adjust the headrest.
- Sit down beside and behind the chair. Is it easy to position yourself close to the chair at each working position?
- Recline in the dental chair and experience the comfort.
- Check the motion. Is it bumpy or smooth?
- Notice if the equipment feels sturdy.
- Open and close the cabinets. Do they open and close smoothly?
- Check that all parts fit well together.
- Test the function of the delivery system, chair, and all components. Just as you wouldn't buy a car without taking it for a test drive, don't buy dental equipment without testing its functionality.

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1.2 Performance and efficiency

The ultimate test of any equipment is how well it performs. Each piece should be ergonomically designed not only for patient comfort, but for the dentist's comfort as well

1.2.1 Reduce Fatigue

Treatment teams have to work in a neutral position, which implies that the more they move, the more energy they waste. The tenser the muscles, the less efficiently they operate. Through the years, the daily aches and pains of poor positioning accumulate and can lead to chronic injury, severely impacting ability to practice dentistry. Look for equipment that will reduce the motion and make every move count.

1.2.2 Ergonomics

The environment must be ergonomically designed. By eliminating time wasting movements, such as overreaching for an instrument, twisting the body or craning the neck, one can complete procedures more efficiently.

1.3 Reputation and Service

Excessive maintenance can result in costly service calls and lost productivity. Select products that need the least amount of maintenance and service. Choose a manufacturer that has a history of creating innovative products that are durable, reliable, and easy to maintain.

- What brand do they use?
- Would they buy that brand again?
- Who do they trust?

Make sure to choose a manufacturer that stands by its products and will still be in business when it comes time for servicing or upgrading five, six, even ten years down the road.

1.4 Value and Pricing

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Always purchase equipment as a feature-for-feature or dollar-for-dollar experience, one may end up with a mismatch in terms of lasting quality and satisfaction. The whole is worth more than the sum of the parts. Is there really a difference in quality? Such is the difference between value and price. Guide the purchasing decision by:

- Asking questions about product performance, reliability, and durability.
- Researching the manufacturer's products, consistency, and longevity in the marketplace.
- Asking what differentiates a manufacturer's products from the competition and/or new generations of product.
- Learning about the partnership between the manufacturer and your full-service dealer. Do they have an established reputation for standing behind the product? What is their customer service record?
- Defining what you need and expect from your next dental equipment purchase.[1-2]

Fig: 1. Schematic Representation of a Dental Chair [3]

2. Used Methodology

The concept of multiple functionality decision making has been expansively applied in real life decision situations such as management science, engineering, military research, public administration, professional journals and conferences of diversified disciplines. [4-6] Multiple functionality decision making (MCDM) is a suitable method for the selection of most appropriate company and selecting their performance based on quantitative functionality (economical) as well as qualitative functionality (relationship closeness, market reputation etc.). In many practical problems, value of a certain alternatives is usually difficult to judge precisely; instead, they can be expressed through linguistic judgement [7] such as 'poor', 'good', 'excellent' and so on. Atanassov introduced the concept of intuitionistic fuzzy sets (IFS), as a generalization of fuzzy sets, which is capable of capturing the information that includes some degree of hesitation and applicable in various fields of research. In decision making problems, particularly in the case of sales analysis, new product marketing, financial services, etc. there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object. Therefore, in various medical applications, intuitionistic fuzzy sets techniques have been more popular than fuzzy sets techniques in recent years. It has been used to build soft decision making models that can accommodate imprecise information and analyze the extent of agreement in a group of experts. Feasibility and effectiveness of IFSs are illustrated in its applications of decision making by many researchers such as in [8-11] The concept of intuitionistic trapezoidal fuzzy numbers (ITFNs) was introduced by Wang and it may be noted that intuitionistic trapezoidal fuzzy numbers (ITFNs) express more flexible and abundant information than trapezoidal fuzzy numbers

A. Preliminaries

In this section, we describe the basic aspects of intuitionistic fuzzy sets (IFSs) and intuitionistic trapezoidal fuzzy numbers (IFNs), which is well known in literature.

Definition 1.1 Atanassov's [12-13] intuitionistic fuzzy set (IFS) over a finite non empty fixed set X, is a set $\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x) > | x \epsilon X\}$ which assigns to each element $x \epsilon X$ to the set $\tilde{A}$, which is subset of X having the degree of membership $\mu_{\tilde{A}}(x): X \to [0,1]$ and degree of non-membership $\gamma_{\tilde{A}}(x): X \to [0,1]$ satisfying $0 \leq \mu_{\tilde{A}}(x) + \gamma_{\tilde{A}}(x) \leq 1$, for all $x \in X$. For each IFS in X, a hesitation margin $\pi_{A}(x)$, which is the intuitionistic fuzzy index of element x in the IFS $\tilde{A}$, defined by $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$, denotes a measure of non-determinacy. We denote $\mathscr{A}(X)$ the set of all the IFSs on X.

Definition 1.2 Intuitionistic trapezoidal fuzzy number (ITFN) $\tilde{x} = \{(a, b, c, d); \mu_{\tilde{x}}, \gamma_{\tilde{x}}\}$ is a special intuitionistic fuzzy set, whose membership function and non-membership function have been defined as follows:

$$\mu_{\tilde{x}}(x) = \begin{cases} \left(\frac{x-a}{b-a}\right)\mu_{\tilde{x}} & \text{if } a \leq x \leq b, \\ \mu_{\tilde{x}} & \text{if } b \leq x \leq c, \\ \frac{d-x}{d-c} \times \mu_{\tilde{x}} & \text{if } c < x \leq d, \end{cases}$$

$$\gamma_{\tilde{x}}(x) = \begin{cases} \frac{(b-x) + \gamma_{\tilde{x}}(x-a)}{b-a} & \text{if } a1 \leq x \leq b, \\ \gamma_{\tilde{x}} & \text{if } b \leq x \leq c, \\ \frac{(x-c) + \gamma_{\tilde{x}}(d-x)}{d-c}\mu_{\tilde{x}} & \text{if} c < x \leq d, \end{cases}$$

Where $0 \leq \mu_{\tilde{x}} \leq 1$ and $0 \leq \gamma_{\tilde{x}} \leq 1$. Also, $\mu_{\tilde{x}} + \gamma_{\tilde{x}} \leq 1$ for all $a, b, c, d \in R$. The values $\mu_{\tilde{x}}$ and $\gamma_{\tilde{x}}$ represent the maximum membership degree and minimum non-membership degree, respectively.

Definition 1.3 Let $\tilde{X}_1 = \{(a_1, b_1, c_1, d_1, \mu_{\tilde{x}1}, \gamma_{\tilde{x}1})\}$ and $\tilde{X}_2 = \{(a_2, b_2, c_2, d_2, \mu_{\tilde{x}2}, \gamma_{\tilde{x}2})\}$ be two trapezoidal intuitionistic fuzzy numbers and δ is a real number. Some basic arithmetical operations (addition, multiplication etc.) are defined as follows

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$$\tilde{X}_1 \oplus \tilde{X}_2 = \{(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2);\ \mu_{\tilde{x}1} + \mu_{\tilde{x}2} - \mu_{\tilde{x}1}\mu_{\tilde{x}2}, \gamma_{\tilde{x}1}\cdot\gamma_{\tilde{x}2}\}$$

$$\tilde{X}_1 \odot \tilde{X}_2 = \{(a_1.a_2, b_1.b_2, c_1.c_2, d_1.d_2);\ \mu_{\tilde{x}1}\cdot\mu_{\tilde{x}2}, \gamma_{\tilde{x}1} + \gamma_{\tilde{x}2} - \gamma_{\tilde{x}1}\cdot\gamma_{\tilde{x}2}\}$$

$$\delta_{\tilde{x}1} = \{(\delta a_1, \delta b_1, \delta c_1, \delta d_1); 1-(1-\mu_{\tilde{x}1})^{\delta}, \gamma_{\tilde{x}1}^{\delta}\}$$

Definition 1.4 intuitionistic trapezoidal fuzzy ideal solutions is defined as

$$\tilde{I}^+ = \{(a^+, b^+, c^+, d^+,); \mu^+, \gamma^+\} = \{(1,1,1,1); 1,0\}$$

Definition 1.5 (Normalized Hamming Distance)
Let $\tilde{X}_1 = \{(a_1, b_1, c_1, d_1); \mu_{\tilde{x}_1}, \gamma_{\tilde{x}_1}\}$ and $\tilde{X}_2 = \{(a_2, b_2, c_2, d_2); \mu_{\tilde{x}_2}, \gamma_{\tilde{x}_2}\}$ be two intuitionistic trapezoidal fuzzy numbers. The normalized hamming distance between $\tilde{X}_1$ and $\tilde{X}_2$ is defined as

$$d(\tilde{x}_1, \tilde{x}_2) = \frac{1}{8}\left\{\begin{array}{l} |(1+\mu_{\tilde{x}_1} - \gamma_{\tilde{x}_1})a_1 - (1+\mu_{\tilde{x}_2} - \gamma_{\tilde{x}_2})a_2| \\ +|(1+\mu_{\tilde{x}_1} - \gamma_{\tilde{x}_1})b_1 - (1+\mu_{\tilde{x}_2} - \gamma_{\tilde{x}_2})b_2| \\ +|(1+\mu_{\tilde{x}_1} - \gamma_{\tilde{x}_1})c_1 - (1+\mu_{\tilde{x}_2} - \gamma_{\tilde{x}_2})c_2| \\ +|(1+\mu_{\tilde{x}_1} - \gamma_{\tilde{x}_1})d_1 - (1+\mu_{\tilde{x}_2} - \gamma_{\tilde{x}_2})d_2| \end{array}\right\}$$

The main aim of this paper is to study intuitionistic trapezoidal fuzzy multiple functionality decision making (ITF-MCDM) problem for evaluating the best alternative whose information take the form of ITFNs and weights of the attributes are completely unknown. The procedure of evaluating the weights of the attributes has been described in section II. In section III, a new algorithm for ITF-MCDM problem has been proposed for evaluating the best alternative whose information takes the form of ITFNs and the ranking of the alternatives has been determined.

3. Method for Evaluating Weights of Attributes with Itfns

A multiple functionality decision making problem includes a discrete set of m possible alternatives $A = \{A_1, A_2, \ldots, A_m\}$, which is based on a set of n evaluation criterions $C = \{C_1, C_2 \ldots C_n\}$. The intuitionistic trapezoidal fuzzy decision matrix is expressed as $\tilde{D} = [\tilde{r}_{ij}]_{m\times n} = \{[a_{ij}, b_{ij}, c_{ij}, d_{ij}]; \mu_{ij}, \gamma_{ij}\}_{m\times n}$

where $\tilde{r}_{ij}$ is the rating of i^{th} alternative meeting the jth functionality which is jointly provided by the decision makers, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

Weight measure plays an important role in multiple functionality decision making problems and has a direct relationship with the distance measure between two fuzzy numbers. In order to deal with decision information with intuitionistic trapezoidal fuzzy numbers, we use the normalized hamming distance between intuitionistic trapezoidal fuzzy numbers as given in definition 1.5. Let w_j represents the weight vector of j^{th} attribute and let the information about these weights is unknown. However, the weights of the attributes have been provided by the decision maker's qualitative opinion. For the sake of intuitionistic formulation of the qualitative opinions, we define the following table:

Table: 1. Linguistic Variables and ITFNS

Sr. No	Linguistic variables	ITFNs
1	Very Poor	{[0.2,0.3,0.4,0.5]; 0.7,0.1}
2	Poor (P)	{[0.3,0.4,0.5,0.6]; 0.8,0.1}
3	Satisfactory (SF)	{[0.4,0.5,0.6,0.7]; 0.2,0.7}
4	Good (G)	{[0.5,0.6,0.7,0.8]; 0.5,0.4}
5	Very Good (VG)	{[0.6,0.7,0.8,0.9]; 0.7,0.3}

If there are p persons in a decision making committee, who qualitatively define the weights of the n criterions, then the effective weight of each functionality in the form of intuitionistic trapezoidal fuzzy number can be evaluated as:

$$w_j = \frac{1}{p}(\tilde{w}_j^1 + \tilde{w}_j^2 + \cdots \tilde{w}_j^p)$$

If $d(\tilde{w}_j, \tilde{I}^+)$ is distance between the weight ITFN $\tilde{w}_j$ and the intuitionistic trapezoidal fuzzy ideal solution $\tilde{I}^+$ then the distance vector is given by

$$N = [d(\tilde{w}_1, \tilde{I}^+), d(\tilde{w}_2, \tilde{I}^+) \ldots\ldots\ldots d(\tilde{w}_n, \tilde{I}^+)].$$

Further, the normalized distance vector on vector N' is given by

$$N' = [\varepsilon_j] = \left[\frac{d(\tilde{w}_j, \tilde{I}^+)}{(Max)_j, d(\tilde{w}_j, \tilde{I}^+)}\right], \qquad j = 1,2..n$$

The entropy measure of the j^{th} functionality (C_j) for m available alternatives can be obtained from:

$$e_j = -\frac{1}{\ln(m)}\left[\frac{\varepsilon_j}{\sum_{j=1}^{n}\varepsilon_j}\ln\left(\frac{\varepsilon_j}{\sum_{j=1}^{n}\varepsilon_j}\right)\right]$$

Finally, the crisp value of weight for jth criterion, which is based on the above entropy measure, can be calculated as follows

$$w_j = \frac{1 - e_j}{n - \sum_{k=1}^{n} e_k} \qquad j = 1,2 \ldots\ldots n$$

4. Algorithm for Intuitionistic Trapezoidal

Fuzzy Multiple Functionality Decision Making

The ranking procedure for a discrete set of m possible alternatives based on a set of n evaluation functionality in case of intuitionistic trapezoidal fuzzy multi functionality decision making (ITF-MCDM) problem is given below:

Input A discrete set of m possible alternatives $A = \{A_1, A_2, \ldots, A_m\}$, a set of n evaluation criterions $C = \{C_1, C_2 \ldots C_n\}$ and weights of functionality in terms of qualitative opinions of decision makers.

· **Step 1:** If there are p persons in a decision making committee, then construct the decision matrix D by calculating the rating of each alternative meeting the functionality as follows:

$$\tilde{r}_{ij} = \frac{1}{p}(\tilde{r}_{ij}^1 + \tilde{r}_{ij}^2 \ldots + \tilde{r}_{ij}^p)$$

Step 2: Since the information about the weights of attributes is unknown, we find the attribute weights using the entropy method as discussed in section II.

Step 3: Make use of definition 1.5 and the obtained weight vector in step 2 to compute the distances $d(A_j, \tilde{I}^+)$ for each □ as follows:

$$d(A_j, \tilde{I}^+) = \sum_{j=1}^{n} w_j\, d(\tilde{I}^+, \tilde{r}_{ij})$$

Step 4: Finally, the most important step to rank the alternatives is performed using the values of the distances

$(A_i, \tilde{I}^{+})$ where $i = 1, 2 \ldots m$. The basic idea of ranking the alternatives used is - smaller the value of $d(A_i, \tilde{I}^{+})$ better the performance/closeness of an alternative to intuitionistic trapezoidal fuzzy ideal solution.

Case study:

Let suppose one dental clinic want to purchase one dental chair. There are three companies {C1, C2, C3}which are providing machine. Among them one machine has to be purchased. There are three experts {E1, E2, E3} who have knowledge about dental chair. Following functions {F1, F2, F3, F_4, F_5} are considered by expert:

I. Durability and reliability
II. Performance and efficiency
III. ergonomics
IV. Reputation and service
V. Value and pricing

Table: 2. Experts Rating Variables

Criteria/experts rating	E_1	E_2	E_3
Durability and reliability (C_1)	VG	SF	SF
Performance and efficiency (C_2)	G	P	G
Ergonomics (C_3)	VG	G	SF
Reputation and service (C_4)	G	G	VG
Value and pricing(C_5)	SF	VP	VP

Table: 3. Experts Rating to Various Functions

Functions	**F_1**	**F_2**	**F_3**
C_1	[.57,.67,.77,.87]; .19,.01	[.43,.53,.63,.73]; .23,.06	[.40,.50,.60,.70]; .31,.01
C_2	[.43,.53,.63,.73]; .32,.01	[.47,.57,.67,.87]; .27,.11	[.53,.63,.73,.83]; .31,.02
C_3	[.47,.57,.67,.87]; .27,.11	[.47,.57,.67,.87]; .27,.11	[.43,.53,.63,.73]; .23,.06
C_4	[.43,.53,.63,.73]; .23,.06	[.57,.67,.77,.87]; .19,.01	[.43,.53,.63,.73]; .23,.06
C_5	[.33,.43,.53,.63]; .32,.00	[.47,.57,.67,.87]; .27,.11	[.50,.60,.70,.80]; .29,.03

The rating of the alternatives with respect to various functions as given by the experts is provided in the following table IV

Table: 4. Experts Ratings for Various Firms

CRITERA	**SUPPLIER FIRM**	**E_1**	**E_2**	**E_3**
C_1	F_1	G	VG	VG
	F_2	SF	SF	G
	F_3	G	P	SF
C_2	F_1	P	G	G
	F_2	VG	SF	VG
	F_3	VG	SF	VG
C_3	F_1	G	SF	G
	F_2	SF	G	G
	F_3	SF	SF	G
C_4	F_1	G	SF	SF
	F_2	VG	G	VG
	F_3	SF	G	SF
C_5	F_1	P	SF	P
	F_2	G	SF	G
	F_3	G	VG	SF

In order to solve the problem, we first evaluate the weights of each criterion with the help of pre-defined linguistic variables in the form of ITFNs and tabulate them in the following

Table: 5. Normalized Decision Matrix

FUNCTIONS	WEIGHT
C_1	{[0.47,0.57,0.67,0.77] ; 0.27,0.05}
C_2	{[0.43,0.53,0.63,0.73] ; 0.32,0.01}
C_3	{[0.5,0.6,0.7,0.8] ; 0.29,0.03}
C_4	{[0.53,0.63,0.73,0.83] ; 0.31,0.02}
C_5	{[0.27,0.37,0.47,0.57] ; 0.31,0.002}

Based on the normalized decision matrix given in table V, the attribute weights can be calculated by using the entropy method with ITFNs:

Table V.

The computed weights are as follows:

$w_1 = 0.19987$; $w_2 = 0.2000$; $w_3 = 0.2015$; $w_4 = 0.2032$; and $w_5 = 0.1954$. It may be noted that

$$\sum_i w_i = 0.9999 \approx 1.$$

By using the weight vector, we get the distances

$(F_1, \tilde{I}^{+}) = 0.63576$

$(F_2, \tilde{I}^{+}) = 0.59069$

$(F_3, \tilde{I}^{+}) = 0.6240$

Finally, based on the idea of ranking given in step 4 of the algorithm, we conclude that desirable order of selecting a company is

$F_2 > F_3 > F_1$

5. Conclusions

The study of multiple functionality decision making (MCDM) problems for evaluating the best alternative has been done with the concept of intuitionistic trapezoidal fuzzy numbers (ITFNs). This algorithm for ITF-MCDM problem has been proposed where the weights of the involved attributes are unknown. On the basis of the decision maker's qualitative opinion to the attributes with the help of pre-defined linguisticterms and an entropy measure, these weights have been calculated. Finally, the selection on the basis of ranking of the company has been done by calculating the hamming distance between the ideal alternative and all the available alternatives.

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