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Image compression by discrete wavelet transforms using global thresholding

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Abstract
Signal **c**ompression is the process of converting data files into smaller files for efficiency of storage and transmission. Image compression is a method through which we can reduce the storage space of images which is helpful to increase storage and transmission process's performance**.** The image compression is implemented in software using MATLAB's Wavelet Toolbox, two dimensional discrete wavelet transform technique. The study is carried out on .jpg format images. We describe the comparison of performance of discrete wavelets like Haar and biorthogonal wavelets. This paper includes the discussion on principles of image compression, image compression methodology, the basics of wavelet and wavelet transforms, the selection of discrete wavelet transform with results and conclusion.

1. Introduction

The Fourier transform splits a signal into frequencies, and measures the amplitude and alignment of each frequency. The Fourier transform has ability to measures spatial frequencies in the image. If you imagine horizontal or vertical bars of color repeating at different speeds, these are the frequencies that the Fourier transform is measuring. In 1965, a new algorithm called Fast Fourier Transform (FFT) was developed and FT became even more popular. The Fourier transform is powerful tools for processing signals that are composed of some combination of sine and cosine signals (sinusoids) [1-2].The Fourier transform shows up in a remarkable number of areas outside of classic signal processing. But this has limited ability to study non - stationary signals. Fourier transform is a useful tool to study the stationary time series, but this analysis is not adapted to non- stationary time series [3]. Grossman and Morlet [4] developed the wavelet transform and Meyer demonstrated the condition for orthogonality of this new mathematical operator. The wavelet word indicates a set of functions, which is called mother wavelet.

1.1Wavelet

A wavelet is a wave-like oscillation with an amplitude that begins at zero, increases, and then decreases back to zero(figure-1).It is a waveform of effectively limited duration that has an average value of zero. Wavelets are functions defined over a finite interval and having an average value of zero. Wavelet transform decomposes a signal into a set of basic functions (wavelets).The ability of wavelets to provide good time resolution. Wavelet transforms are well localized in both time and frequency domain. Wavelets often give a better signal representation using Multi-resolution analysis. Wavelets also allow filters to be constructed for stationary and non-stationary signals. The wavelets have some properties: have good time-frequency (time-scale) localization, can represent data parsimoniously, can be implemented with very fast algorithms and are well suited for building mathematical models of data. The wavelet transform approach of signal analysis is also flexible in handling irregular data sets.

Wavelets are obtained from a single prototype wavelet ψ(t) called mother wavelet by dilations and shifting:

$$\psi_{a,b} = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) \quad (1)$$

Where, a -the scaling parameter and b is the shifting parameter.

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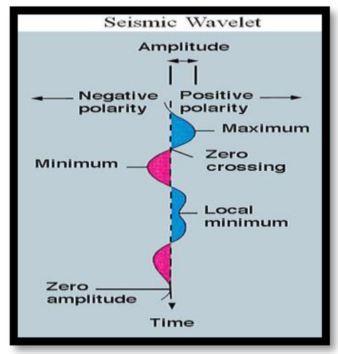

Fig.1: Seismic wavelet

Wavelet transform has recently become a very popular when it comes to analysis, de-noising and compression of signals and images . The wavelet analysis depends on the type of the wavelet used, its level of decomposition and threshold value. The approximation signals describe pixel values of image and detail signal describe the horizontal, vertical, and diagonal details of an image. Threshold depends on the information which we obtained from the detail signals. Higher compression ratio is the main goal of image compression; higher compression ratio can be also obtained through the selection of threshold values. If the information obtained from detail signals are very small then it is set zero. Higher compression can be achieved when the number of zeros is greater. The image is said to be loss less if the energy retained is 100% then the compression is known as "lossless compression''. This occurs when the threshold value is set to zero, meaning that the detail has not been changed. If any values are changed then energy will be lost and this is known as "lossy'' compression.

2. Discrete wavelet transforms and multi-resolution analysis

This transform decomposes the signal into mutually orthogonal set of wavelets. The discrete wavelet transform provides sufficient information both for analysis and synthesis of the original signal, with a significant reduction in the computation time. The DWT is considerably easier to implement when compared to CWT. In the discrete wavelet transform, an image signal can be analyzed by passing it through an analysis filter bank followed by a decimation operation. This analysis filter bank**,** which consists of a low pass and a high pass filter at each decomposition stage, is commonly used in image compression. When a signal passes through these filters, it is split into two

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bands. The low pass filter, which corresponds to an averaging operation, extracts the coarse information of the signal. The high pass filter, which corresponds to a differencing operation, extracts the detail information of the signal. The output of the filtering operations is then decimated by two. A three-level decomposition is shown in figure 2(a, b &c).

Wavelets are a special kind of functions which exhibits oscillatory behavior for a short period of time and then die out[1]. For any two real numbers *a* and *b*, a wavelet function is as given by eq (1):

If we choose a=2-j and b/a =k, we get discrete wavelet.

ψ j, k (t) = 2j / 2 ψ (2j t - k)

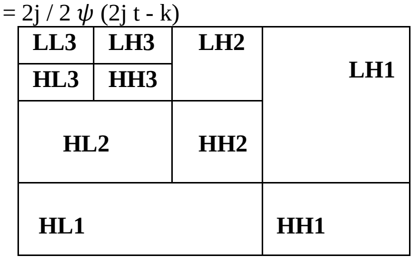


Fig2(a)

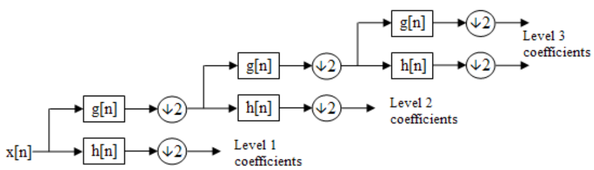


Fig 2(b)

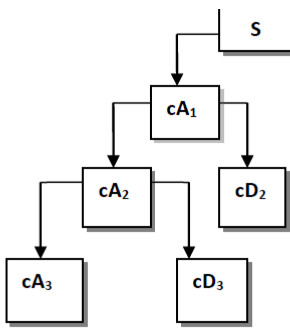


Fig2(c)

Fig 2: Three level Decomposition

The wavelet transform of a signal captures the localized time frequency information of the signal. A multi resolution analysis (MRA) [11] is a radically new recursive method for performing discrete wavelet analysis. A MRA for introduced by Mallat [10] and extended by other researchers consists of a Sequence V_j: jϵZ of closed subspaces of $L^2(R)$, aspace of square integrable functions, satisfying the following properties;

1. $V_{j+1} \subset V_j$: $j \in \mathbb{Z}$
2. $\cap_{j\in\mathbb{Z}} V_j = \{0\}$, $\overline{\cup_{j\in\mathbb{Z}} = L^2(\mathbb{R})}$,
3. For every, $L^2(\mathbb{R})$, f (t) $\in V_j \Rightarrow f\left(\frac{t}{2}\right) \in V_{j+1}$, $\forall j \in \mathbb{Z}$
4. There exists a function $\phi(t) \in V_o$ such that $\{\phi(t - k): k \in \mathbb{Z}\}$ is orthonormal basis of V_o.

The function ϕ (t) is called scaling function of given MRA and property 3 implies a dilation equation.

$$\phi(t) = \sum_{k\in Z} h_k \ \phi(2t - k)$$

Where h_k is low pass filter and is defined as:

$$h_k = \left(\frac{1}{\sqrt{2}}\right) \int_{-\infty}^{\infty} \phi(t)\ \phi(2t - k)\, dt$$

Now we consider W_o be orthogonal compliment of V_o in V_1i.e.

$V_1 = V_o \oplus W_o$

If $\psi \in W_o$ be any function then,

$$\psi(t) = \sum_{k\in Z} h_k \ \phi(2t - k)$$

Where, $g_k = (-1)^{k+1}\ h_{1-k}$ are high pass filters.

We can express a signal in terms of bases of V_0space and W_0space. If we combine the bases of V_0 and W_0 space, we can express any signal in V_0 space.

Using the same argument, we can write

$V_2 = V_1 \oplus W_1$

In general,

$V_j = V_{j-1} \oplus W_{j-1}$

But, $V_{j-1} = V_{j-2} \oplus W_{j-2}$

Therefore

$V_j = W_{j-1} \oplus W_{j-2} \oplus V_{j-2}$

.. ..

$V_j = W_{j-1} \oplus W_{j-2} \oplus W_{j-3} \oplus \ldots\ldots\ldots\ldots \oplus W_0 \oplus V_0$

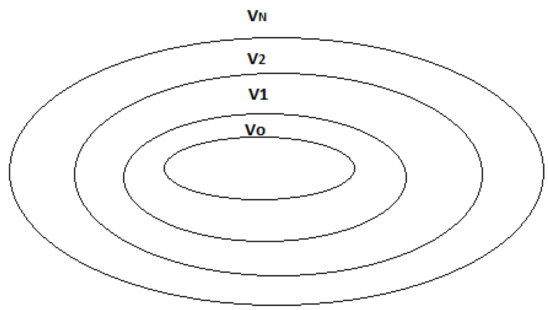
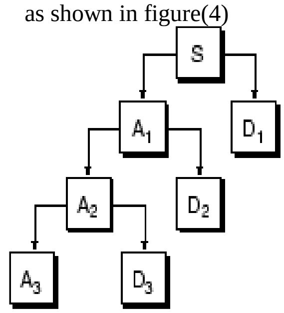


Fig 3: Space Signal

Let S= {S_n: n∈ ℤ} be a function sampled at regular time interval, $\Delta t = \tau$ where ℤ is an integer. S is split into a "blurred" version a_1 at the coarser interval$\Delta t = 2\tau$ and "detail" d_1 at scale $\Delta t = \tau$.This process is repeated and gives a sequence S, a_1, a_2, a_3, a_4,…..of more and more blurred versions together with the details d_1, d_2, d_3, …….removed at every scale ($\Delta t = 2^m \tau$ in a_m and d_{m-1}).

Here a_m's and d_m's are approximation and details of original signal. After N iteration S can be reconstructed as $S = a_N + d_1 + d_2 + d_3 + d_4 + d_5 + \ldots + d_N$. The approximations are the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components. Thus the original signal, S, passes through two complementary filters in which one is low pass filter and second one is high pass filter as shown in figure(4)



$$S = A_1 + D_1$$

$$= A_2 + D_2 + D_1$$

$$= A_3 + D_3 + D_2 + D_1$$

Fig. 4: Pass Filter

2.1 Haar

The Haar transform is one of the oldest transform functions, proposed in 1910 by the Hungarian mathematician Alfred Haar. Haar described the first wavelet basis in 1910Haar wavelet is discontinuous, and resembles a step function. It represents the same wavelet as daubechies1 [5].The Haar wavelet's mother wavelet function ψ (t) can be described as:

$$\psi(t) = \begin{cases} 1 & 0 \le t < \frac{1}{2} \\ -1 & \frac{1}{2} \le t < 1 \\ 0 & otherwise \end{cases}$$

Its scaling function $\phi(t)$ can be described as:

$$\phi(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & otherwise \end{cases}$$

It conserves the energy of signal and compaction of the energy of signals. In discrete form, Haar wavelets are related to a mathematical operation called the Haar transform. Due to its low computing requirements, the Haar transform has been mainly used for image processing and pattern recognition. From this reason two dimensional signal processing is an area of efficient applications of Haar transform due their wavelet like structure. It is found effective in applications such as signal and image compression in electrical and computer engineering as it provides a simple and computationally efficient approach for analyzing the local aspects of a signal. It shows orthogonal, biorthogonal and compact support. Its dwt as well as cwt also possible and scaling and wavelet function is shown in figure 5(a) and 5(b).

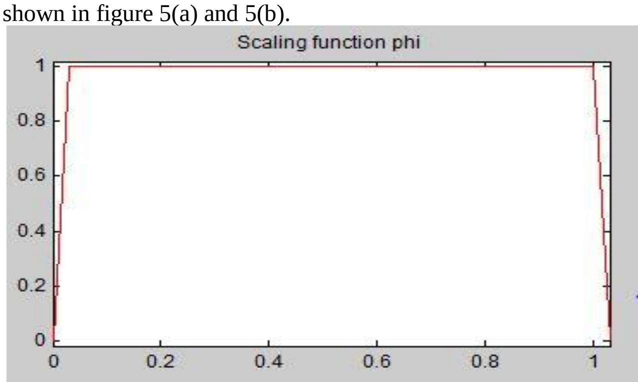


Fig.5(a) Scaling function

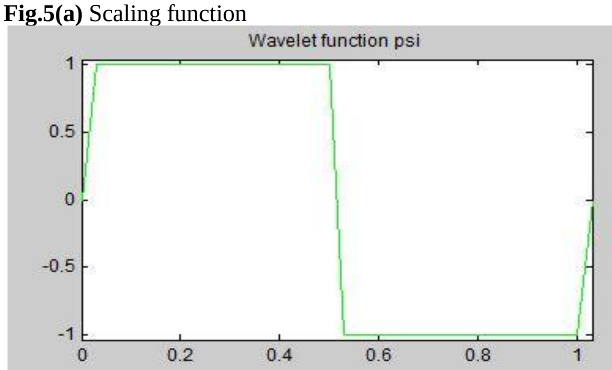


Fig. 5(b) Wavelet function

2.2 Bi-orthogonal wavelet

This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction. By using two wavelets, one for decomposition (on the left side) and the other for reconstruction (on the right side) which is shown in figure instead of the same single one, interesting properties are derived. The decomposition scaling and wavelet function is shown in figure 6(a) and 6(b).

Bi-orthogonal Wavelets [2] are families of compactly supported symmetric wavelets. The symmetry of the filter coefficients is often desirable since it results in linear phase of the transfer function. In the bi-orthogonal case, rather than having one scaling and wavelet function, there are two scaling functionsϕ and ϕ* that may generate different multi resolution analysis, and accordingly two different wavelet functions ψ and ψ*.

ψ *is used in the analysis and ψ is used in the synthesis

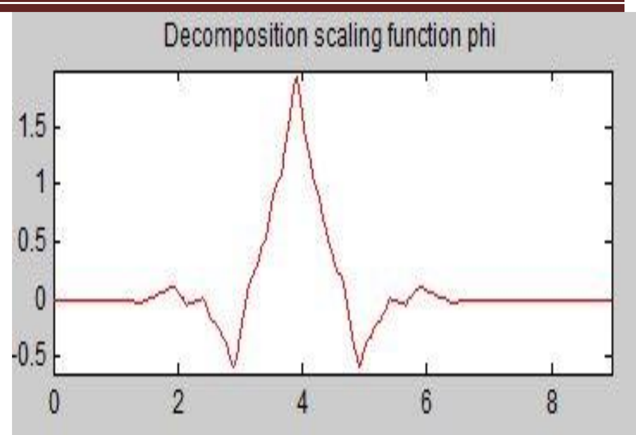


Fig.6 (a): Decomposition scaling function

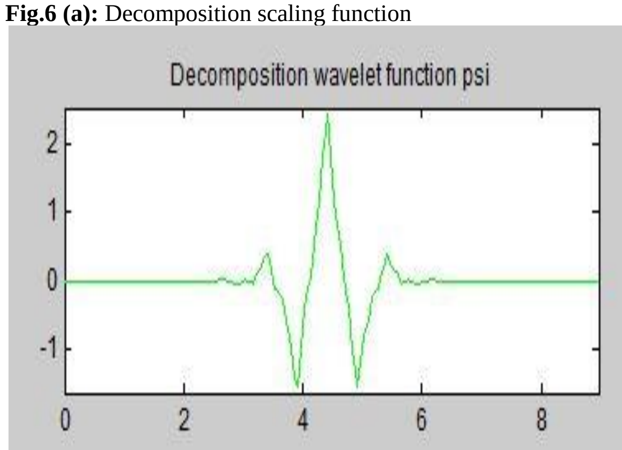


Fig. 6(b): Decomposition wavelet function

3. Principle of Image Compression

Image compression is minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level. The reduction in file size allows more images to be stored in a given amount of disk or memory space. It also reduces the time required for images to be sent over the Internet or downloaded from Web pages. Compression ratio [7] is defined as ratio of the size of original data set to the size of the size of compressed data set.

$$\text{Compression Percentage} = \frac{C-D}{C} \text{x}100 \qquad (2)$$

Where C = Number of bytes in the original data set
D= Number of bytes in the compressed data set.

3.1 Global Thresholding

Thresholding [12] is a process of converting a gray scale input image to a bi-level image by using an optimal threshold. In global thresholding, a single threshold for all the image pixels is used. When the pixel values of the components and that of background are fairly consistent in their respective values over the entire image, global thresholding could be used. Global thresholding consists of setting an intensity value (threshold) such that all voxels having intensity value below the threshold belong to one phase; the remainder belong to the other. Global thresholding is as good as the degree of intensity separation between the two peaks in the image. It is an unsophisticated segmentation choice. In thresholding we select the maximum value in array and calculate the threshold value by:

$$T_o = |X_{max}| / 2 \qquad (3)$$

The value which are greater than T_o is significant and others are insignificant form zero tree root [6]. By choosing a particular intensity value as "threshold", images can be segmented by setting those pixels whose original intensity is above the threshold as "white pixels", and setting the other

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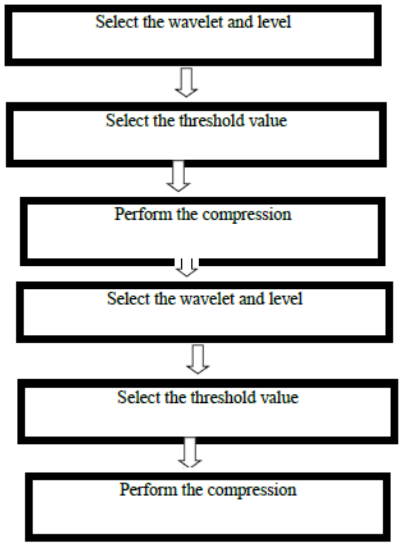
pixels as "black pixels." Thresholding is one of the easiest methods to automatically segment an image using a computer.

3.2 Methodology



4. Results

In this paper, we compared Biorthogonal and Haar wavelet of Discrete wavelet transform (DWT). We compared the performance of these transforms on image Baby.jpg(512x512). As in above discussion more percentage of zeros might be responsible for more compression, and high value of the energy retained shows the less loss of the information. In Haar wavelet retained energy is 98.92% and in biorthogonal transform it is 99.18%.Also compression percentage is more for bior transform.

Hence, bior transform is better for image compression.

Figure 7 is original image. Compressed image by biorthogonal transform and their residual and retained energy figure are on left side.Compressed image by haar transform and also their resiual and retained energy graph are on right side.Graph shows the retained energy and zeros in %.

Fig. 7: Original Image

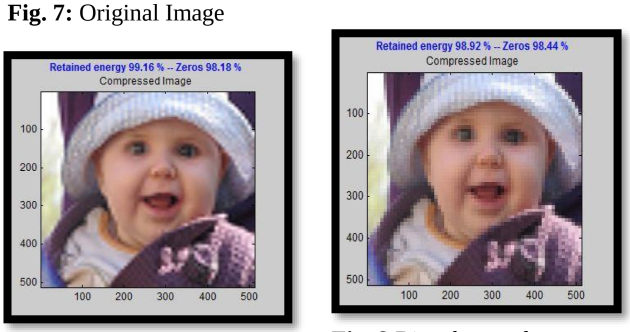


Fig. 8 Bi-orthogonal Transform

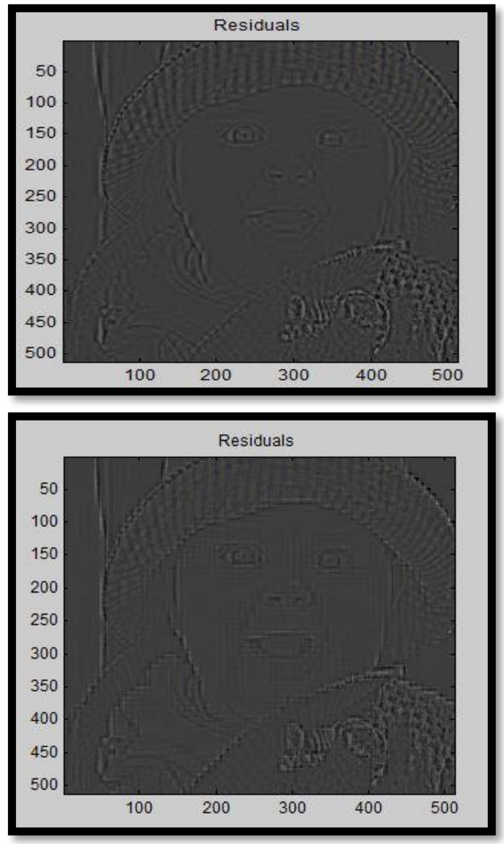


Fig 9 Haar Transform

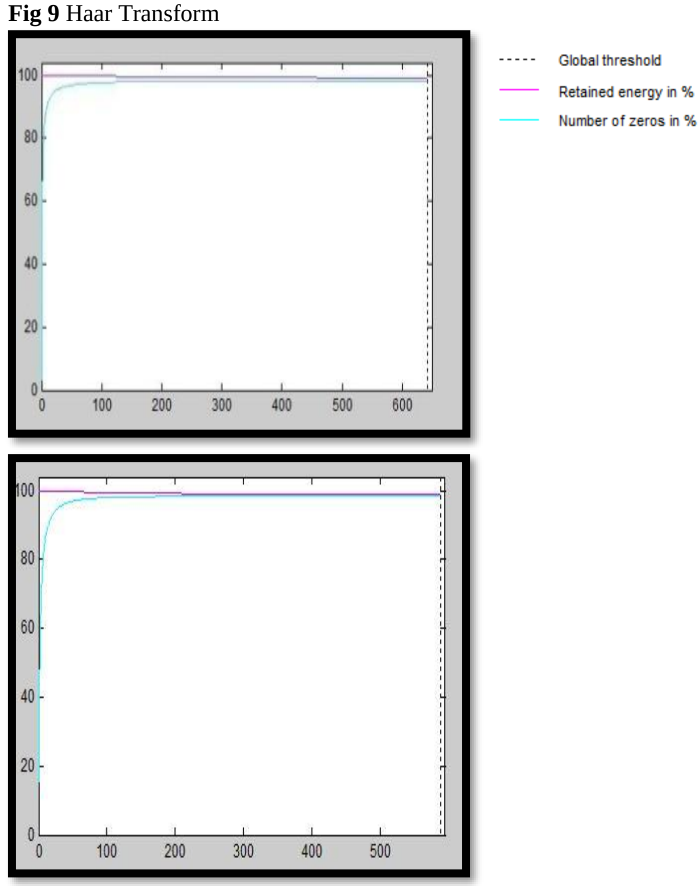


Fig.10 Retained Energy

Table 1: Image change data

S.No.	Wavelet	Size of original image in bytes	Size of compressed image in bytes	Compression %
1.	Biorthogonal	44,825	25,015	44.19%
2.	Haar	44,825	27,226	38.14%

References

[1.] A Kumar, S Kumar, JK Pathak. Spectral Analysis of River Ramganga Hydraulics using Discrete wavelet transform, International Conference of Advance Research and Innovation, 2015, 370-373

[2.] BBS Kumar, PS Satyanarayana. Image Analysis Using Biorthogonal Wavelet, International Journal of Innovative Research and Development, 2(6), 2013, 543-565.

[3.] M Frigo, SG Johnson. The design and implementation of FFTWJ, proceedings of the IEEE, 93(2), 2005, 216- 231.

[4.] A Grossmann, J Morlet, T Paul. Transforms associated to square integrable group representation, J. Math. Phys. 26, 1985, 2473-2579.

[5.] I Daubechies. The wavelet transform, time frequency localization and signal analysis, IEEE Trans. Inform, 1990.

[6.] JM Shapiro. Embedded Image coding using zero trees of wavelet coefficient, IEEE Transaction in signal processing, 41(12), 1993, 3445-3462.

[7.] GK Kharate, VH Patil, NL Bhale. Selection of Mother Wavelet for Image Compression on Basis of Nature of Image, Journal of Multimedia, 2(6), 2007, 44-51.

[8.] CR Gonzalez, E Richard. Woods, Steven L. Eddins – Digital Image Processing Using MATLAB, 1- 78, 256-295 & 296-547, 1st Edition 2006.

[9.] R Pratap. Getting started with MATLAB7, 1-15, 17-44 & 49-79, 2nd Edition 2006.

[10.] SG Mallat. A wavelet tour of Signal Processing, Academic Press, New York. 1998.

[11.] SG Mallat. A theory for multi resolution signal decomposition: the wavelet representation, 674-693, 1989.

[12.] V Alarcon-Aquino, JM Ramirez-Cortes, P Gomez-Gil, O Starostenko, H Lobato-Morales. Lossy Image Compression Using Discrete Wavelet Transform and Thresholding Techniques, The Open Cybernetics & Systemics Journal, (7), 2013, 32-38.