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Mathematical analysis on blood flow in stenotic inclined artery

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Abstract
In the present paper, blood is considered as an electrically conducting Newtonian fluid. To estimate the effect of the stenosis shape, a suitable geometry has been considered such that the horizontal shape of the stenosis can easily be changed just by varying a parameter referred to as the shape parameter, which analyzed mathematically of MHD blood flow in a stenotic inclined artery. The problem is described by the usual MHD equations along with appropriate boundary conditions which is consistent with the principles of magnetohydrodynamics. The effects of various parameters associated with the present flow problem such as radially variable viscosity, stenotic shape and pulsatile Reynolds number on the physiologically important flow characteristics namely velocity distribution, flow rate and wall shear stress have also been analyzed.

1. Introduction

In developed and developing countries, one of the major health hazards is arteriosclerosis. Arteriosclerosis or stenosis means the abnormal and unnatural growth in the lumen of an artery that develops at various locations in the cardiovascular system under unfavorable conditions. One of the major factor of stenosis is to deposits of cholesterol on the arterial wall and proliferation of connective tissues may be reasonable for the same [1,18].

Several researchers have studied blood flow under different flow situations in tapered tubes [17]. Experimental work on blood flow in tapered tubes[6,15-16]. Flow of blood (Casson fluid) in tapered tubes is dealt in Oka [8]. Walawender *et al.* have obtained simplified solutions for limiting case of very slow and very fast flows for a non-Newtonian fluid in a uniformly tapering tube.

The cosine-shaped geometry was considered and analyzed with different parameters by many researchers like Young [18], Kapur [4], Chakravarty [2]. The power-law and Casson fluid models with cosine-shaped geometry were discussed by Shukla *et al*. [10]. A composite shaped geometry of arterial stenosis was suggested and investigated by Mekheimer [5]. Misra and Shit [7] discussed the bell-shaped geometry with different fluids. In the above studies the shape of stenosis was considered to be symmetrical about the axis as well as radius of the flow cylinder.The radially non-symmetric stenosis has been analyzed by Sanyal and Maji [9], Srivastava and Saxena [14], Srivastava [15]. The effects of shape of stenosis on the resistance to blood flow through an artery has been investigated by Haldar [3]. Due to the presence of a new parameter the formulation of our analysis is mathematically more general and includes the model of Haldar [3].

In the present paper, the author deals with a problem in which blood flow has been considered through an artery with mild stenosis and the effects of various parameters associated with the present flow problem such as radially variable viscosity, hematocrit, plasma layer thickness and pulsatile Reynolds number on the physiologically important flow characteristics namely velocity distribution, flow rate and wall shear stress have been analysed.

2. Problem Formulation

In the present analysis, we have considered the axisymmetric flow of blood in a uniform circular artery with an axially non symmetric but radially symmetric mild stenosis. The geometry of the stenosis is shown in fig.1. For different values of shape parameter m, the asymmetric shapes of the stenosis are sketched in fig 2.

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The geometry of the asymmetric shapes of the stenosis in the arterial segment is mathematically represented by the following equation:

$$h(z) = \frac{R(z)}{R_0} = \begin{cases} 1 - A[L_0^{m-1}(z-d) - (z-d)^m] & d \le z \le L_0 \\ 1 & otherwise \end{cases} \quad (1)$$

Where
R(z) : Radius of stenotic artery .
R(0) : Radius of normal artery.
L : The length of the artery.
L0 : The length of the stenosis.
d : Distance between equispaced points.
m : Parameter determining the shape of stenosis ($m \ge 2$).

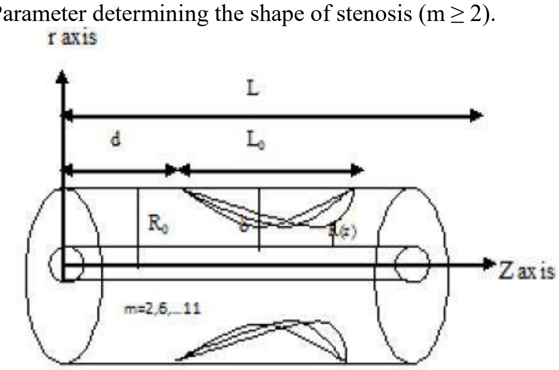

Fig-1: Geometry of an axially non-symmetric stenosis in an artery

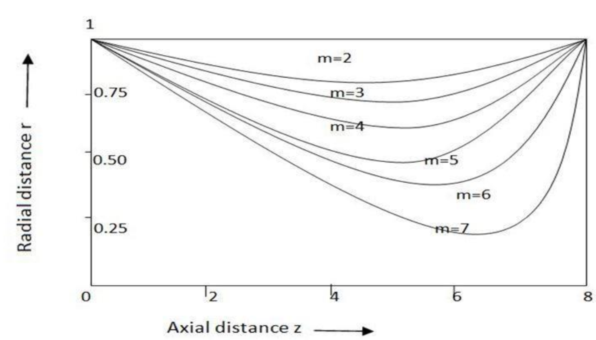

Fig-2: Shape of the arterial stenosis for different values of the stenosis shape parameter.

The parameter A is defined by

$$A = \frac{\delta}{R_0 L_0^m} \frac{m^{\frac{m}{m-1}}}{m-1}. \quad (2)$$

$$z = d + \frac{L_0}{m^{\frac{m}{m-1}}}, \text{ such that } \frac{\delta}{R_0} \le 1 \quad (3)$$

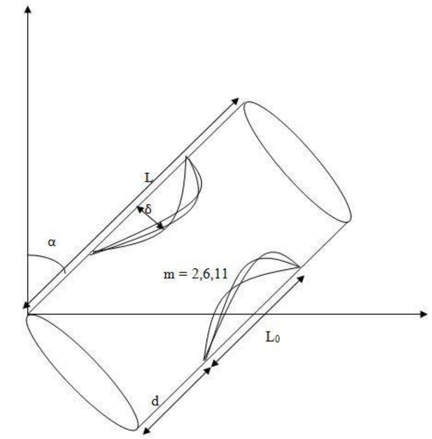

Fig-3:Geometry of an inclined stenosed artery with axially non-symmetrical stenosis

Here, the body fluid blood is assumed to behave as a Newtonian fluid (Schlitchting and Gerstein 2004) [3]. The equations (as obtained from Navier-Stokes equations of motion for viscous fluids) describing the steady flow of Newtonian fluid are given by

$$0 = \frac{\partial \bar{p}}{\partial \bar{r}}. \qquad (4)$$

$$0 = \frac{\partial \bar{p}}{\partial \bar{\theta}}. \qquad (5)$$

$$0 = \bar{\rho}\,\bar{g}\cos\alpha - \frac{d\bar{p}}{d\bar{z}} - \frac{1}{\bar{r}}\frac{\partial(\bar{r}\,\bar{\tau}_{rz})}{\partial \bar{r}}. \qquad (6)$$

Also $\bar{\tau}_{rz} = -\bar{\mu}\frac{\partial \bar{u}}{\partial \bar{r}}$ (7)

is the expression for shear stress at a radial distance r from the axis, $\bar{u} = \bar{u}(r)$ is the axial component of velocity and $\bar{\mu}$ is the shear viscosity for Newtonian fluid.

In view of equation (7), equations (4) to (6) takes the form

$$\frac{d\bar{p}}{d\bar{z}} = \frac{\bar{\mu}}{\bar{r}}\frac{\partial\left(\bar{r}\frac{\partial \bar{u}}{\partial \bar{r}}\right)}{\partial \bar{r}} + \bar{\rho}\,\bar{g}\cos\alpha. \qquad (8)$$

Boundary conditions for given problem can be taken as follows:

$$\bar{u} = \bar{u}_s \quad \text{at } \bar{r} = \bar{R}(\bar{z}). \qquad (9a)$$

$$\frac{\partial \bar{u}}{\partial \bar{r}} = 0 \quad \text{at } \bar{r} = 0. \qquad (9b)$$

Where, $\bar{u}_s$ is the slip velocity at the stenotic wall.

Now, introduce the following non dimensional variables

$$z = \frac{\bar{z}}{R_0}, \quad R = \frac{\bar{R}}{R_0}, \quad \mu = \frac{\bar{\mu}}{\mu_0}, \quad \frac{dp}{dz} = \frac{d\bar{p}/d\bar{z}}{C_0}, \quad u = \frac{\bar{u}}{u_0}. \qquad (10)$$

$$4\frac{dp}{dz} = \frac{\mu}{r}\frac{\partial\left(r\frac{\partial u}{\partial r}\right)}{\partial r} + \frac{4\cos\alpha}{F}, \quad F = \frac{\bar{\tau}_0}{\bar{\rho}\,\bar{g}}. \qquad (11)$$

Boundary conditions 9(a, b) in the dimension less form becomes

$$u(r) = u_s \qquad \text{at } r = R. \qquad (12a)$$

$$\frac{\partial u}{\partial r} = 0 \qquad \text{at } r = 0. \qquad (12b)$$

The volumetric flow rate Q is defined by

$$Q = 4\int_{r=0}^{R} urdr \qquad (13)$$

The expression for the wall shear stress is given by

$$\tau = -\frac{1}{2}\mu\frac{\partial u}{\partial r}. \qquad (14)$$

3. Solutions

The solution of the equation of motion can be written as

$$u = -\frac{r^2}{\mu}\left[\frac{\cos\alpha}{F} + C\right] + C_1. \qquad C = -\frac{dp}{dz}.$$

Using boundary condition 12(a, b) we obtained

$$u = u_s + \frac{(R^2 - r^2)}{\mu}\left[\frac{\cos\alpha}{F} + C\right] + C_1. \qquad (15)$$

Using above value of u in equation (13) the volumetric flow rate obtained as below

$$Q = R^2\left[\frac{1}{\mu}\left(C + \frac{\cos\alpha}{F}\right)R^2 + 2u_s\right]. \qquad (16)$$

and pressure gradient is given by

$$\frac{dp}{dz} = \mu[2R^2u_s - Q]R^4 + \frac{\cos\alpha}{F}. \qquad (17)$$

The wall shear stress at r = R(z) takes the form

$$\tau_r = R\left(C + \frac{\cos\alpha}{F}\right). \qquad (18)$$

4. Results and Discussions

By the inspection of the figures-4 and figures-5, it is revealed that the wall shear stress has the minimum value at the extremities of stenosis, then it starts increasing with stenosis height along the axial distance of the artery and attains the maximum value at the stenosis throats and it goes on decreasing to the minimum value and it attain its maximum magnitude at very near the termination for asymmetric stenosis (m = 6,11). Figure 5 represents the variation of the wall shear stress with stenosis shape parameter. The wall shear stress shows same variations for the equidistant values of *z* from the extremities of the constriction when *m* = 2 which indicates the radially non-symmetric stenosis becomes symmetric when *m* = 2. The behavior of τr (in Fig. 6) with variations in parameters m and C are show in figure-6. It indicates that as C increases (0.5, 1.5), τr increases, also as shape parameter m increases (= 2,6,11) τr decreases, after attaining the maximum magnitude at throat of stenosis for symmetric region (m = 2) and that at near the termination for (m = 6,11).

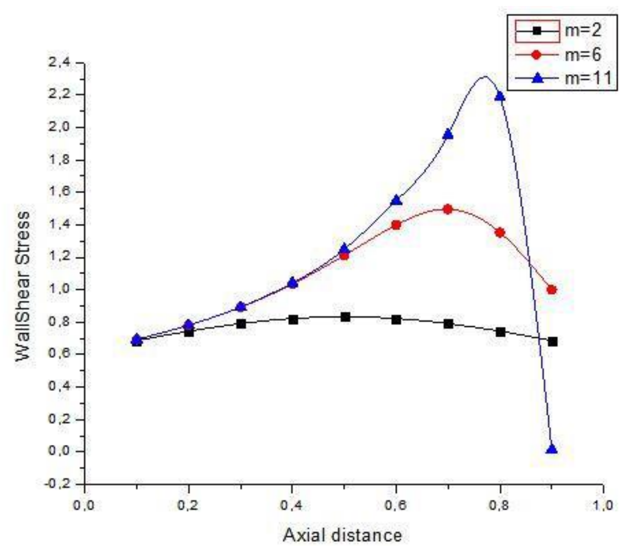

Fig. 4: Variation of wall shear stress with axial distance for α= 900, C =0.5 different values of m

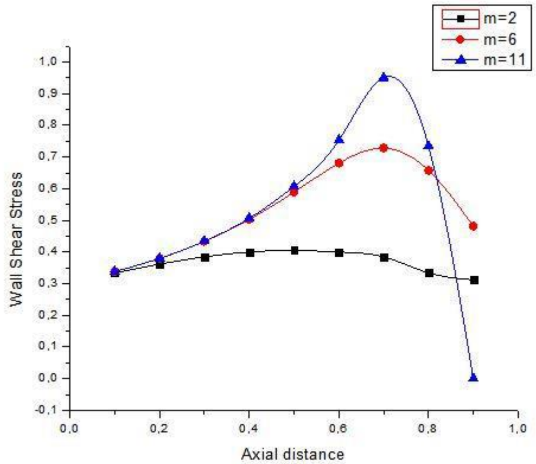

Fig-5: Variation of wall shear stress with axial distance for α= 600, C =0.5 different values of m

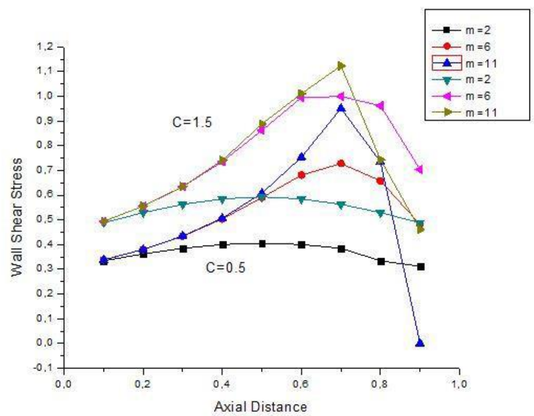

Fig-6: Variation of wall shear stress with axial distance for different values of C and α= 600

5. Conclusions

In the present work, MHD blood flow in a stenotic inclined artery has been investigated. The volumetric flow rate Q is given by the equation (16). Wall shear stress increases as stenosis grows and stenosis shape parameter increases. Also it is observed that, increase in shape parameter, increases the wall shear stress in the upstream of the throat but decreases in the downstream. In this investigation, it is easily noticed that consideration of an inclined artery and asymmetric stenosis artery, could provide better results than that of a non-inclined artery. We hope that this investigation may be useful for medical practitioners to understand the blood flow through arterial system.

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