ISSN 2347 - 3258
International Journal of Advance Research and Innovation
National Conference on Modern Trends in Analytical & Environmental Physics (Hindu College, Moradabad)

Signal Denoising of 1-D Signal with Optimal Base Wavelet

Anil Kumar
Department of Physics, Hindu College, Moradabad-244001 (UP)

Article Info
Article history:
Received 02 April 2017
Received in revised form
20 May 2017
Accepted 28 May 2017
Available online 15 June 2017

Keywords: Wavelet, Discrete Wavelet Transforms, Shanon Entropy, Signal Denoising.

Abstract
Signal denoising is the process of removing noise from a signal for efficiency of getting information. The Signal denoising is implemented in software using MATLAB's Wavelet Toolbox. The study is carried out on 1-D bump signal. First of all we select the optimal base wavelet for the given signal. Thereafter Shanon Entropy cost function thresholding is applied. This paper includes the discussion on the basics of wavelet, discrete wavelet transforms, selection of optimal wavelet, Shanon entropy cost function thresholding, signal denoising with results and conclusion.

1. Introduction

Wavelet is a new development in the emerging field of data analysis for Physicists, Engineers, and Environmentalists [1, 2, 4]. It represents an efficient computational algorithm under the interest of a broad community. Fourier sine‟s extracts only frequency information from a time signal, thus losing time information [7], while wavelet extracts both time evolution and frequency composition of a signal. Wavelet is a special kind of the functions which exhibits oscillatory behavior for a short time interval and then dies out. In wavelet we use a single function and its dilation and translation to generate a set of ortho normal basis functions to represent a signal. Number of such functions is infinite and we choose one that suits to our application. The range of interval over which scaling function and wavelet function are defined is known as support of wavelet. Beyond this interval (support) the functions should be identically zero. There is an interesting relation between length of support and number of coefficients in the refinement relation. For orthogonal wavelet system, the length of support is always less than no. of coefficients in the refinement relation. It is also very helpful to require that the mother function have a certain number of zero moments, according to:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

The mother function can be used to generate a whole family of wavelets by translating and scaling the mother wavelet.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) = T_b D_a \psi$$

Here b is the translation parameter and a is the dilation or scaling parameter. Provided that ψ(t) is real-valued, this collection of wavelets can be used as an orthonormal basis. A critical sampling of the continuous wavelet transform is

$$W_{a,b} = \int f(t)\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)dt$$

is obtained via $a = 2^{-j}$, where j and k are integers representing the set of discrete translations and discrete dilations. Upon this substitution, we can write discrete wavelet transform as;

$$W_{j,k} = \int f(t)2^{j/2}\psi(2^j t - k)\,dt$$

Wavelet coefficients for every (a, b) combination whereas in discrete wavelet transform, we find wavelet coefficients only at very few points by the dots and the wavelets that follow these values are given by

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$$

These wavelet coefficient for all j and k produce an orthonormal basis. We call $\psi_{0,0}(t) = \psi(t)$ as mother wavelet. Other wavelets are produced by translation and dilation of mothere wavelet. The wavelet transform of a signal captures the localized time frequency information of the signal. Suppose we are given a signal or sequences of data $S = \{S_n\}_{n\in\mathbb{Z}}$ sampled at regular time interval □t □□□. S is split into a "blurred" version a1 at the coarser interval □t □□2□□and "detail" d1 at scale □t □□□. This process is repeated and gives a sequence S_n , a_1, a_2, a_3, a_4,…..of more and more blurred versions together with the details d_1, d_2, d_3, d_4, …….removed at every scale ($\Delta t = 2^m\tau$ in a_m and d_m). Here a_ms and d_ms are approximation and details of original signal. After N iteration the original signal S can be reconstructed as

$$S = a_N + d_1 + d_2 + d_3 + \cdots \ldots.. d_N$$

2. Multi resolution Analysis and Signal decomposition & Reconstruction

A multiresolution analysis for $L^2(\mathbb{R})$ introduced by Mallat [9, 10] consists of a Sequence V_j, j∊ℤ of closed subspaces of $L^2(\mathbb{R})$. Let f (x) be a function in $L^2(\mathbb{R})$. We can write f □□x□□in V_{j+1} space, i.e.,□□□

$$f(x) = \sum c_{j+1,k}\,\phi_{j+1,k}(x)$$

Since $V_{j+1} = V_j \oplus W_j$

where $V_{j+1} = \text{span}\,\overline{\left(\phi_{j+1,k}(x)\right)}$

and $V_j = \text{span}\,\overline{\left(\phi_{j,k}(x)\right)}$

$W_j = \text{span}\,\overline{\left(\psi_{j,k}(x)\right)}$

Therefore

$$f(x) = \sum_k c_{j,k}\,\phi_{j,k}(x) + \sum_{j_0}^{j}\sum_k d_{j,k}\,\psi_{j,k}(x)$$

where $c_{j,k} = \langle f, \phi_{j,k}\rangle = \int f(x)\,\phi_{j,k}(x)\,dx, \quad \forall\, k \in \mathbb{Z}$

and $d_{j,k} = \langle f, \psi_{j,k}\rangle = \int f(x)\,\psi_{j,k}(x)\,dx$

are collectively known as approximation and detailed coefficients. Thus given signal takes place a new version such as

$$f(1) = a_1 + d_1$$
$$f(2) = a_2 + d_2 + d_1$$
$$f(3) = a_3 + d_{3+}d_2 + d_1$$
$$f(4) = a_4 + d_4 + d_{3+}d_2 + d_1$$
$$f(5) = a_5 + d_5 + d_4 + d_{3+}d_2 + d_1$$
$$f(6) = a_6 + d_6 + d_5 + d_4 + d_{3+}d_2 + d_1$$
$$f(7) = a_7 + d_7 + d_6 + d_5 + d_4 + d_{3+}d_2 + d_1$$

***Corresponding Author,**
E-mail address: akumarmbd@gmail.com

ISSN 2347 - 3258
International Journal of Advance Research and Innovation
National Conference on Modern Trends in Analytical & Environmental Physics (Hindu College, Moradabad)

$$f(8) = a_8 + d_8 + d_7 + d_6 + d_5 + d_4 + d_3 + d_2 + d_1$$

Here $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are approximations of signal and $d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8$ are details of signal at various scale or time frames. Since maximum 8 level are possible therefore, f(8) will be combined into a zero frequency component a_8 as well as 8 frequency components $d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8$.

$$f(1) = \sum_k c_{1,k}\,\phi_{1,k}(x) + \sum_k d_{1,k}\,\psi_{1,k}(x)$$

$$f(2) = \sum_k c_{2,k}\,\phi_{2,k}(x) + \sum_k d_{2,k}\,\psi_{2,k}(x) + \sum_k d_{1,k}\,\psi_{1,k}(x)$$

□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□
□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□

$$f(8) = \sum_k c_{8,k}\,\phi_{8,k}(x) + \sum_k d_{8,k}\,\psi_{8,k}(x) + \sum_k d_{7,k}\,\psi_{7,k}(x) + \sum_k d_{6,k}\,\psi_{6,k}(x) + \sum_k d_{5,k}\,\psi_{5,k}(x) + \sum_k d_{4,k}\,\psi_{4,k}(x) + \sum_k d_{3,k}\,\psi_{3,k}(x) + \sum_k d_{2,k}\,\psi_{2,k}(x) + \sum_k d_{1,k}\,\psi_{1,k}(x)$$

3. Selection of optimal base wavelet for noised signal

The primary and most important work in the spectral analysis of any signal using wavelet transforms is the selection of suitable wavelet according to the signal [3, 8]. Suitable wavelet is selected on the basis of compatibility with signal characteristics. Accurate wavelet selection retains the original signal and also enhances the frequency spectrum of denoised signal.

Noise is the unwanted, problematic and unavoidable part of signal. A signal is represented as,

$$f = f_* + \sigma.\rho$$

Where, f is the noise corrupted version of signal f_* and σ is the noise level and ρ is unit energy noise process. Here f_* is coherent and ρ is non-coherent with respect to optimal base wavelet.

3.1 Thresholding

Thresholding is a technique performing to zero out small magnitude wavelet coefficients and retain the large magnitude wavelet [5, 6]. Signal noise ratio is the measurement of signal relative to noise and is described in terms of Shannon Entropy [10].

3.2 Coherence

A coherent signal is one that exhibits a concentration of energy in the representation domain and an incoherent signal is one whose energy is diffusely spread throughout the representation domain. A signal is coherent with respect to wavelet if the energy in the inner product representation is concentrated, that is, well localized in the representation domain.

3.3 Shannon Entropy Cost function thresholding

Shannon Entropy is used to measure the amount of uncertainty in a probability distribution. Shannon Entropy Cost function is defined as,

$$M\left(c,\{b_j\}\right) = -\sum_{j=1}^{M} \left|\langle c, b_j\rangle\right|^2 \log\left|\langle c, b_j\rangle\right|^2$$

Where, $M = 2^N$, $1 \le j \le N$, $c \in \mathbb{R}^M$. A best basis relative to M for c is a system $B^* \in B$ for which $M\left(c, B^*\right)$ is minimum.

Here, $$M\left(c,\{b_j\}\right) = \sum_{j:\left|\langle a,b_j\rangle\right| \ge 1} \left|\langle c,b_j\rangle\right|^2$$

is a direct measure of mean square error encountered when the small (meaning below threshold) coefficients are discarded and the signal is reconstructed using the large (above threshold) coefficients.

The signal noise ratio (SNRAT) is measured in decibels and given by

$$SNRAT = -10\log_{10}\left(M\left(c/\|c\|,\{b_j\}\right)\right)$$

For a given threshold value $0 < \lambda$, we define, $M\left(c,\{b_j\}\right) = \left|\left\{n : \left|\langle c,b_j\rangle\right| \ge \lambda\right\}\right|$

In the context of signal processing cost function M measures how many coefficients are negligible (that is below threshold) in a transformed signal and how many are important.

The basis that concentrate the signal energy over a few coefficients, also reveals its time frequency structures, is called best basis. A best wavelet packet basis divides the time frequency plane into elementary atoms that are best adapted to approximate a particular signal. The best basis associated to a signal minimizes the Shannon Entropy function or Cost function M. Finding the minimum M, we require more than $2^{N/2}$ operations, which is computationally prohibitive. The fast dynamic programming algorithm of Coifman & Wickerhauser [4] find best basis with $O\left(N \log_2 N\right)$ operations by taking advantage of the tree structure.

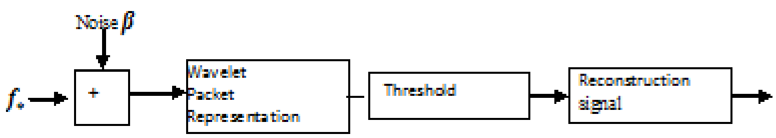

Fig.1: A block diagram for noise suppression and reconstruction algorithms)

4. Results and Discussion

A bumps signal for $N = 8$, so that the length of signal is $2^8 = 256$.

The noise to the above signal with signal noise ratio 1.5412.

The discrete wavelet transform of the above noisy bumps signal using wavelet of Daubechies 4 wavelet, level 4.

Most of the wavelet packet coefficients are nearly zero. Taking threshold value $\lambda = 0.7$ and using optimal base Daubechies 4 wavelet, level 4, we suppress noise and reconstruct the signal.

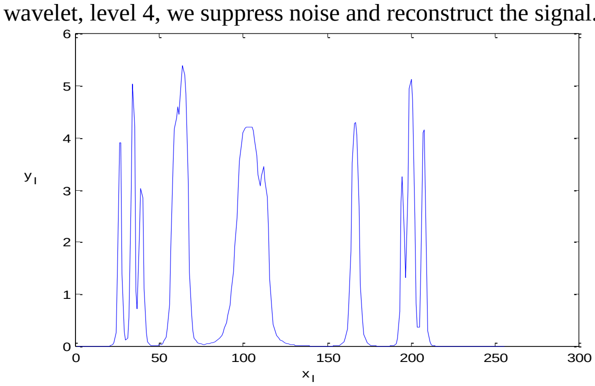

Fig. 2: Bumps signal

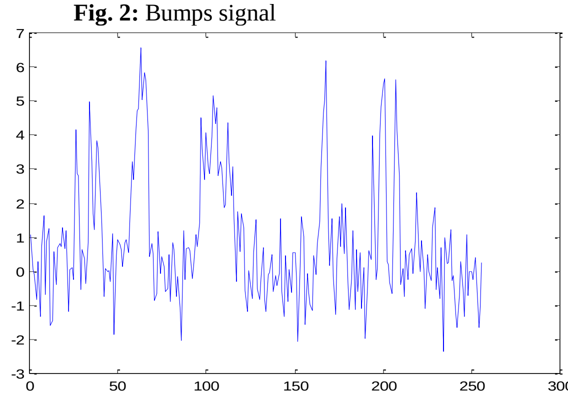

Fig. 3: Noised signal

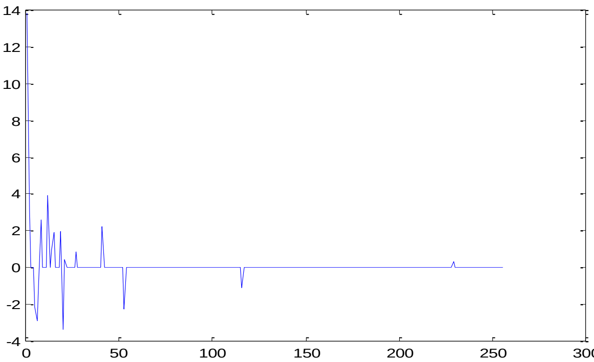

Fig. 4: Wavelet coefficients for noised bumps signal

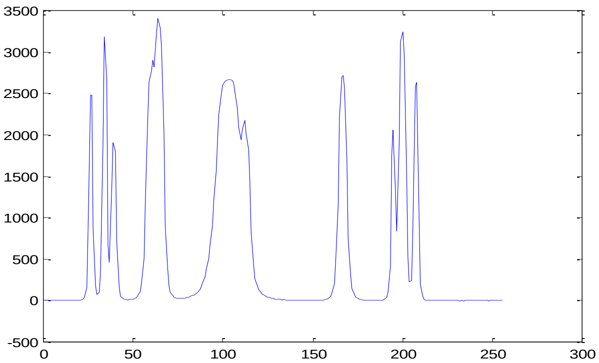

Fig. 5: Denoised signal

5. Conclusions

The structure of discrete wavelet transforms provides a natural tool for denoising. Daubechies4 wavelet is the optimal base wavelet for given bump signal. Our approach of signal denoising is helpful for data compression as well as modulation and demodulation. Quantized coefficients are used to reconstruct a version of input via an appropriate reconstruction algorithm.

Reference

[1.] A Kumar, S Kumar, JK Pathak. Spectral Analysis of River Ramganga Hydraulics using Discrete wavelet transform, International Conference of Advance Research and Innovation, 2015, 370-373.

[2.] CA Guerin, M Holchneider. Wavelet dimension and time evolution, Wavelets in Physics (Van Der Berg, J. C., ed.), 2004, 421-446.

[3.] JJ Benedetto. Irregular sampling and frames: Wavelets**,** A Tutorial in Theory and Applications, C. K. Chui, editor, Academic Press, Boston, 1992, 445-507.

[4.] JP Antoine. Wavelet analysis: A new tool in Physics, Wavelets in Physics (Van Der Berg, J. C., ed.), 2004, 9-21.

[5.] KPK Soman, I Ramchandran. Insight into wavelets: From theory to practice, 2nd ed., Prentice Hall of India,2005.

[6.] A Teolis. Computational signal processing with wavelets**,** in Applied and Numerical Harmonic Analysis, Birkhauser, Boston, 1998, 171-193.

[7.] ND Pah, DK Kumar. Thresholding wavelet network for signal classification, International Journal of Wavelets, Multi resolution and Information Processing, 1(3), 2003, 243-261.

[8.] RJ Duffin, AC Schaeffer. Class of non-harmonic Fourier series, Trans. Amer. Math. Soc. **7**2, 1952, 341-366.

[9.] RR Coifman, Y Meyer, MV Wickerhauser. Wavelet analysis and signal processing”**,** Wavelets and their Applications (MB Ruski et al., eds.), Johen and Bartlet Publishers, Boston, 1992, 153-178.

[10] SG Mallat. A theory for multi resolution signal decomposition: The wavelet representation, IEEE Trans. On Pattern Analysis and Machine Intelligence, 11, 1989, 674-693.

[11] SG Mallat. Multi resolution approximations and wavelet ortho normal bases of $L^2(\mathbb{R})$, Trans. Amer. Math. Soc., 315, 1989, 69-87.