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Dual Solutions for Flow and Heat Transfer of a Non-Newtonian fluid over a shrinking sheet

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Abstract
In the present work, flow and heat transfer of a second grade fluid over a shrinking sheet is studied. Using similarity transformations, the governing boundary layer equations are transformed into non-liner ordinary differential equations. Numerical solution of the problem is obtained by Runge-Kutta method with shooting technique. Numerical results for dimensionless velocity and temperature profiles as well as for the skin friction at the wall and local rate of heat transfer are obtained and displayed through graphs for pertinent parameters. It is found that dual solution exists for a certain range of parameters. It is also observed that to increase the magnitude of viscoelastic parameter the momentum boundary layer thickness as well as thermal boundary layer thickness increases for both upper and lower branch solutions.

1. Introduction

The boundary layer flow over a stretching surface is encountered in several technological processes. Such situations occur in polymer processing, manufacturing of glass sheet, paper production, in textile industries and many others. Sakiadis [1] initiated a study on the boundary layer flow of viscous fluid towards a linear stretching sheet. An exact similarity solution for the dimensionless differential system was obtained. The problem in [1] is extended to discuss the various aspects of flow and heat transfer characteristics by many researchers [2-10].

Very recently VanGorder and Vajravelu [11] found the multiple solutions for hydromagnetic flow of a second grade fluid over a stretching or shrinking sheet. In their study they observed that there cannot be more than two positive solutions for momentum equation over shrinking sheet in the case of second grade fluid. Turkyilmazoglu [12] has been investigated the MHD slip flow characteristics of non-Newtonian flow fluid past a continuously shrinking sheet in the presence of mass transfer. They [12] have implemented a theoretical analysis to explore the domains of existence or non-existence as well as unique or multiple slip flow solutions. Motivated by the works mentioned above, we in this paper to investigate the flow and heat transfer characteristics caused by shrinking sheet immersed in a second grade fluid in magnetic field. The results for the skin friction coefficient, local Nusselt number, velocities profiles as well as the temperature profiles are obtained and discussed through graphs and tables for different values of the governing parameters.

2. Formulation of problem

The steady two-dimensional boundary layer flow of an electrically conducting, non-Newtonian fluid (second grade fluid) past a shrinking sheet coinciding with the plane $y = 0$ is considered. A vertical uniform magnetic field of strength H_0 is imposed along y-axis, which produces magnetic effect in x-direction.

Under the usual boundary layer assumptions, the conservation equation of mass and momentum for the flow and heat transfer equation, in the usual notation given as eq (1)

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_0 H_0^2 u}{\rho}-\kappa_0\left[u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial x\partial y}\frac{\partial u}{\partial x}\right] \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho c_p}\left(T-T_\infty\right) \qquad (3)$$

where u and v are the velocity components in x and y directions respectively, $\sigma 0$ is the electrical conductivity of the fluid, ν is the kinematic coefficient of viscosity, $\kappa 0 = -\alpha 1/\rho$ is the elastic parameter where $\alpha 1$ is the material modulus ($\alpha 1 < 0$ is termed as second-order fluid and $\alpha 1 > 0$ is termed as second-grade fluid [4]), k is the thermal conductivity of the fluid medium, cp is the specific heat, ρ is the fluid density (assumed constant) and Q is the rate of volumetric heat generation/absorption.

The appropriate boundary conditions for velocity and temperature are given by

$$u=u_w(x)=-cx,\ v=v_w(x), T=T_w(x)=T_\infty+A(x)^r \quad \text{at } y=0 \qquad (4.1)$$

$$u\to 0, \frac{\partial u}{\partial y}\to 0 \text{ and } T\to T_\infty \text{ as } y\to\infty \qquad (4.2)$$

where c is a constant rate stretching/shrinking (for stretching $c < 0$ and for shrinking $c > 0$), v_w is the wall mass transfer velocity with $v_w > 0$ for injection and $v_w < 0$ for suction, A is constant, T_w is the wall temperature, T_∞ is the free stream temperature (assumed to be constant), r is the variable wall temperature parameter (for $r = 0$ thermal boundary conditions becomes isothermal).

The equation of continuity is satisfied by stream function ψ. introducing the following similarity transformations [4]

$$\psi = x\sqrt{c\nu}.f(\eta) \qquad (5.1)$$

$$\eta = y\sqrt{c/\nu} \qquad (5.2)$$

$$h(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \qquad (5.3)$$

Momentum equation and energy equation becomes

$$f'''-(f')^2+f.f''=Mf'+k\left[2f'.f'''-f.f''''-(f'')^2\right] \qquad (6)$$

$$h''+\Pr.f.h'-r.\Pr.f'.h+\Pr\beta h=0 \qquad (7)$$

Where, prime denote differentiation with respect to η, $M=\dfrac{\sigma_0 H_0^2}{c\rho}$ is the magnetic parameter, $k=\dfrac{\kappa_0 c}{\nu}$ is the viscoelastic parameter ($k<0$ for second grade fluid), $\beta=\dfrac{Q}{\rho c_p}$ is internal heat generation/absorption parameter and Pr is Prandtl number.

The velocity components are given by

$$u=c.x.f' \qquad (8.1)$$

$$v=-\sqrt{c\nu}\,f \qquad (8.2)$$

The boundary conditions becomes

$$f(0)=s,\ f'(0)=-1,\ h(0)=1 \qquad (9.1)$$

$$f'(\infty)\to 0,\ f''(\infty)\to 0 \text{ and } h(\infty)\to 0 \qquad (9.2)$$

where $s={-v_w}\big/{\sqrt{c\nu}}$ is the mass transfer parameter. $s>0$ ($v_w<0$) corresponds mass suction and $s<0$ ($v_w>0$) corresponds to mass injection. Making use of boundary conditions (9a) and (9b), we can obtain an analytical solution of equation (6) in the form

$$f(\eta)=s-\frac{1}{\alpha}\left(1-e^{-\alpha\eta}\right) \qquad (10)$$

where α is a real positive root of cubic algebraic equation

$$ks\alpha^3-\alpha^2(1+k)+s\alpha+M-1=0 \qquad (11)$$

VanGorder and Vajravelu [11] proved in their analysis that there are no cases in which there will be three solution for physically meaningful parameters of equation (11). They also find the exact solution for momentum equation in some special cases,

Case1- when $s = 0$

In this case we get only one physically meaningful solution

$$f(\eta)=-\frac{1}{\alpha}\left(1-e^{-\alpha\eta}\right) \qquad (12)$$

$$\text{where } \alpha=\sqrt{\frac{1+M}{1+k}} \qquad (13)$$

which is valid for $M\geq 0$ and $-1<k\leq 0$.

Case2- when $k = 0$

In the case of Newtonian fluid we have exact solution as

$$f(\eta)=s-\frac{1}{\alpha}\left(1-e^{-\alpha\eta}\right) \qquad (14)$$

$$\text{where } \alpha=\frac{s\pm\sqrt{s^2-4(1-M)}}{2} \qquad (15)$$

In this case we find two solutions, one solution or no solution as,

1. **When $M = 0$**
   a) $s>2$, two exponential solution.
   b) $s=2$, unique solution
   c) $s<2$, no solution
2. **When $0<M<1$**
   a) $s>0$, with $s^2>4(1-M)$, two exponential solution.
   b) $s>0$, with $s^2=4(1-M)$, unique solution.
3. **When $M = 1$** and $s>0$, unique solution.
4. **When $M > 1$** and $-\infty<s<\infty$, unique solution.

For all other cases α becomes negative and hence no solution exist.

The velocity profile is determined from equation (10) to be

$$f'(\eta)=-e^{-\alpha\eta}$$

The main physical quantities of interest are the skin friction coefficient and the local Nusselt number, which are proportional to the quantities $f''(0)$ and $-\theta'(0)$, respectively. Noting that skin friction parameter is $f''(0)=\alpha$.

2. Numerical Procedure

The set of equations (6) and (7) under the boundary conditions (9a) and (9b) have been solved numerically using Runge-Kutta method with shooting technique. We let

$$f=y_1,\ f'=y_2,\ f''=y_3,\ f'''=y_4,\ h=y_5,\ h'=y_6 \qquad (16)$$

$$y_1'=y_2,\ y_2'=y_3,\ y_3'=y_4,$$

$$y_4'=\frac{1}{ky_1}\left(-y_4-y_1y_3+y_2^2+My_2+2ky_2y_4-ky_3^2\right),\ y_5'=y_6,$$

$$y_6'=-\Pr y_1y_6+r\Pr y_2y_5-\Pr\beta y_5 \qquad (17)$$

subject to the following initial conditions

$$y_1(0)=s,\ y_2(0)=-1,\ y_3(0)=t_1,\ y_4(0)=t_2,\ y_5(0)=1,$$

$$y_6(0)=t_3 \qquad (18)$$

To solve this system of equations we require six initial conditions whilst we have only two initial conditions $f(0)$, $f'(0)$ on $f(\eta)$ and one initial condition $\theta(0)$ on $\theta(\eta)$. In shooting method, the unspecified initial conditions $y_3(0)$, $y_4(0)$ and $y_6(0)$ in equation (18) are assumed as t_1, t_2,t_3 respectively, Equation (17) is then integrated numerically as an initial valued problem at η_∞. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at η_∞ with its given value there. If a difference exists, improved value of the missing initial conditions must be obtained and the process is repeated. A step size of $\Delta\eta$ = 0.001was selected to be satisfactory for convergence criteria of 10^{-7} in nearly all cases. The computations were done by a written program which uses a symbolic and computational computer language MATLAB.

3. Result and discussion

Numerical solutions of the governing ordinary differential equations (6) and (7) with the boundary conditions (9) are obtained by Runge-Kutta method with shooting technique. The effect of mass transfer parameter (s), Magnetic Parameter (M), Viscoelastic parameter (k), Prandtl number (Pr), variable wall temperature parameter (r) and internal heat generation/absorption parameter (β) on the velocity $f'(\eta)$, the shear stress at the wall $f''(0)$, temperature field h(η) and local rate of heat transfer at the surface or local Nusselt number (-h'(0)) are shown in Figs. 1-9.

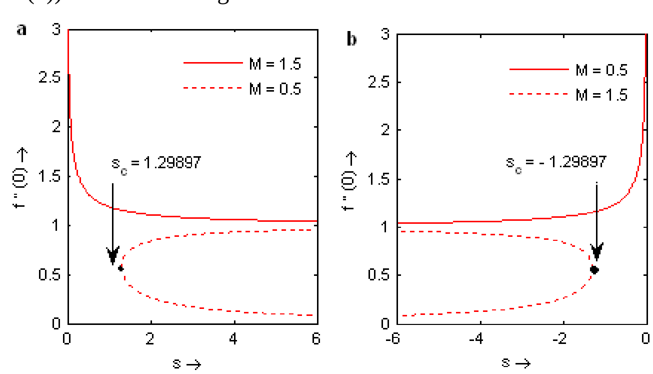


Fig. 1: Skin friction coefficient $f''(0)$ versus the mass suction parameter s when k = -1

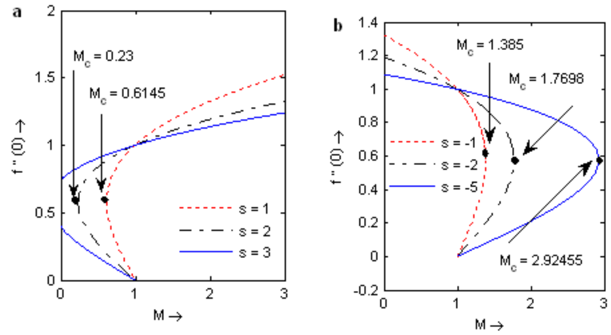


Fig. 2: Skin friction coefficient $f''(0)$ versus the magnetic parameter M when k = -1.

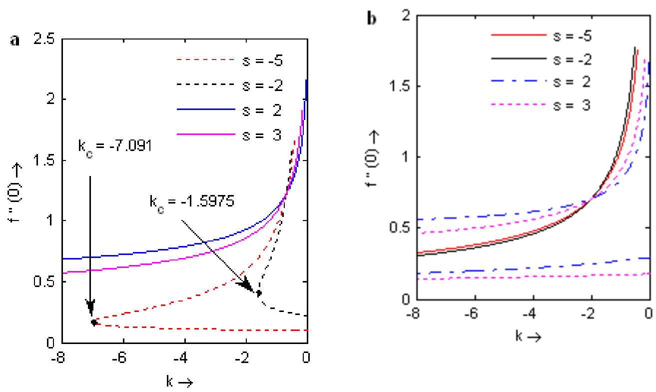


Fig. 3: Skin friction coefficient $f''(0)$ versus the viscoelastic parameter k when (a) M = 1.5, (b) M = 0.5.

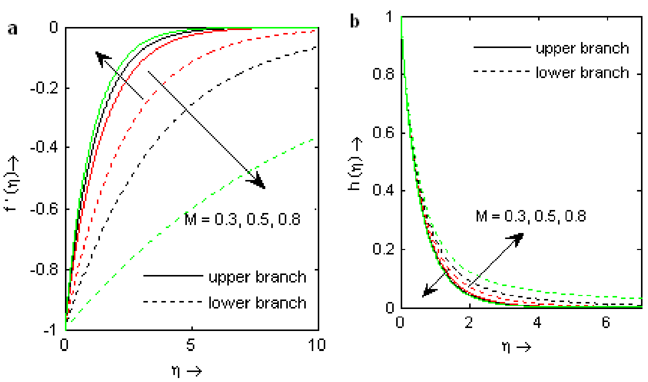


Fig 4: Dual velocity profiles $f'(\eta)$ and Dual temperature profiles h (η) for various values of M with k = -1, s = 2, r = 0, Pr = 1, β = -0.5

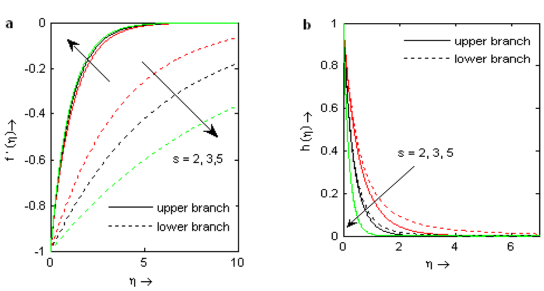
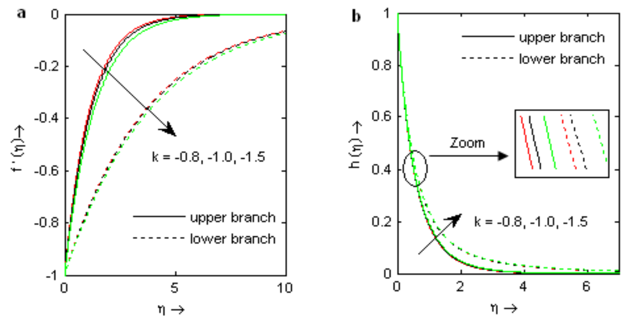


Fig 5: Dual velocity profiles $f'(\eta)$ and Dual temperature profiles h (η) for various values of s with k = -1, M = 0.5, r = 0, Pr = 1, β = -0.5

Fig 6: Dual velocity profiles $f'(\eta)$ and Dual temperature profiles h (η) for various values of k with s = 2, M = 0.5, r = 0,



Pr = 1, β = -0.5

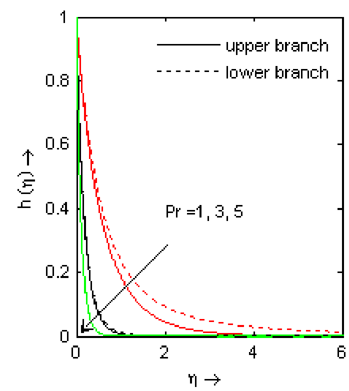


Fig. 7 Dual temperature profiles h (η) for various values of Pr with s = 2, M = 0.5, k = -1, r = 0, β = -0.5

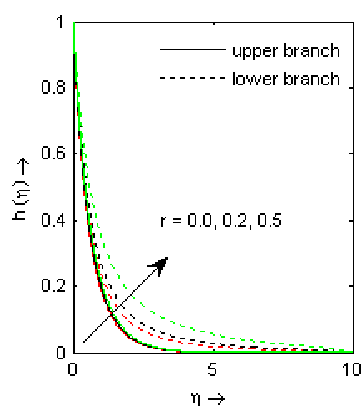


Fig. 8: Dual temperature profiles h (η) for various values of r with s = 2, M = 0.5, k = -1, Pr= 1, β = -0.5

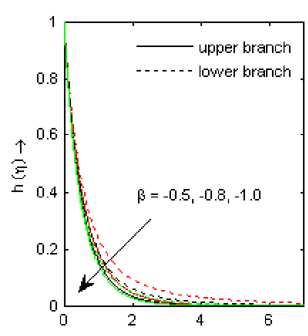


Fig. 9: Dual temperature profiles h (η) for various values of β with s = 2, M = 0.5, k = -1, r = 0, Pr =

4. Conclusions

The following conclusions have been be extracted from this analysis

1) Velocity of boundary layer thickness and thermal boundary layer thickness for first solution is always thinner than that of second solution.
2) The dual solutions for velocity fields and temperature fields are obtained for some values of parameters.
3) Increase in the magnitude of viscoelastic parameter, $|k|$, increase the velocity and thermal boundary layer thickness for upper and lower solution branches.
4) Increasing the wall temperature parameter r also increase the thermal boundary layer thickness.

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