ISSN 2347 - 3258
International Journal of Advance Research and Innovation
National Conference on Modern Trends in Analytical & Environmental Physics (Hindu College, Moradabad)

Analysis of CR-Submanifolds of Nearly and Product Kahlerian Manifold

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Article Info
Article history:
Received 02 April 2017
Received in revised form
20 May 2017
Accepted 28 May 2017
Available online 15 June 2017

Keywords: Kahlerian manifold, Kahlerian product manifold, geodesic CR-sub manifold, holomorphic submanifold

Abstract
Motivated by efficient utilization of manifolds, we initiate the study of CR-sub manifolds of a nearly and Product Kahlerian manifold in the present paper. We carry out their fundamental properties which are necessary elements of further deductions. We deduce the conditions under which the distribution necessary for CR-sub manifolds are integral. We also evaluate mixed totally geodesic CR-sub manifolds. Further, we provide a basic study of Kahlerian product manifold. We evaluate Riemannian curvature tensor, Gauss's equation and curvature tensor of M under the light of basic properties of Kahlerian pro duct manifold. Lastly, we derive totally real sub manifold of product Kahlerian manifold by implementing some restriction on differentiable distribution.

1. Introduction

Various researchers have analyzed CR-submaifolds of nearly Kahlerian and Product Kahlerian manifold. Earlier studies on nearly Kahlerian manifolds are provided by Gray (1969,1970), Takamatsu (1971), Watanabe and Takamatsu (1973) and Chen (1975). Bejancu (1978) derived various aspects of CR-submanifolds of a nearly Kahlerian manifold. Chen (1981) also discussed conditions required for integrable CR-submanifolds of nearly Kahlerian manifold.

The initial study on Kahlerian product manifold was provided by Bejancu (1978). Chen (1981) described various fruitful results of CR-submanifolds of Kahlerian product manifold. Yano and Kon (1981) expanded the study on CR-submanifolds of such manifold.

Ignatochkina and Kirichenko (1999) studied various conformally invariant properties of nearly Kahlerian manifolds.

Borthwick and Uribe (2000) verified the properties of the asymptotic geometry of the maps. Belgun and Moroianu (2001) considered a complete six-dimensional nearly Kahler manifold together with the first canonical Hermitian connection. Banaru (2002) facilitated various results related with the type number of a nearly cosymplectic hypersurfaces in nearly Kahlerian manifolds. Moroianu et al. (2005) provided the facts on unit killing vector fields on nearly Kahlerian manifolds.

Recently, Nagy and Semmelmann (2008) focussed on the study of deformations in nearly Kahlerian manifolds. Ornea (2008) discussed about conformally Einstein products and nearly Kahlerian manifolds.

2. Basic Fundamental Properties

First of all, we discuss about the basic concept of a nearly Kahlerian manifold.

A nearly Kahlerian manifold $\overline{M}$ of dimension *2n* is an almost Hermitian manifold ($\overline{M}$, *G, j*) satisfying.

$$(\overline{\nabla}_X J)(v)^* + (\overline{\nabla}_Y J)(X) = 0, X, Y \in \chi(\overline{M}) \qquad (1)$$

Where $\chi(\overline{M})$ denotes the Lie-algebra of vector fields on $(\overline{M})$.

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Here G implies Hermitian metric. J is an almost complex structure and $\overline{\nabla}$ defines about Riemannian connection.

The general result about Kahler manifold is that every Kahler manifold is a nearly Kahler manifold but the converse of its is not true in general conditions.

If we want the satisfaction of converse that the most important condition is that the almost complex structure should be integrable.

2.1 Example of nearly Kahler manifold

S^6 with cannonical almost structure is a best example of nearly Kahler manifold which is not Kahler. The mathematician Gray exposed that Like Kahler manifolds and nearly Kahler manifolds have rich geometric and topological properties.

Ricci tensor (Ric of $\overline{M}$) is given by the formula

$$\overline{R}ic(X,Y) = \sum_{i=1}^{n} \left[\overline{R}(X, JY, Je_i, e_i) + 2G((\overline{\nabla}_X J)(e_i)(\overline{\nabla}_Y J)(e_i)) \right]$$

(2)

Where $\{e_1, e_2 .... e_n; Je_1, Je_2, ... Je_n\}$ be an Ortho normal frame of nearly Kahler manifold $\overline{M}$, and $\overline{R}$ be the Riemannian curvature tensor. If $\overline{M}$ be a constant holomorphic sectional curvature C then

$$\overline{R}(X,Y,Z,W) = \frac{C}{4}[G(X,W)G(Y,Z) - G(X,Z,G(Y,W) + G(X,JW)GY,JZ) - G(X,JZ)G(Y,JW) - 2G(X,JY)G(Z,JW)) + \frac{1}{4}[G((\overline{\nabla}_X J)(W)(\overline{\nabla}_Y J)Z)) - G((\overline{\nabla}_X J)(Z),(\overline{\nabla}_Y J)(W) - 2G((\overline{\nabla}_X J)(Y),(\overline{\nabla}_Z J)(W) \qquad (3)$$

The sectional curvature is given by

$$\overline{R}(X,Y;X,Y) = \frac{C}{4}\left[G(X,Y)^2 - G(X,X)G(Y,Y) - 3G(X,JY)^2\right] - \frac{3}{4}\|(\overline{\nabla}_X J)(Y)\|^2 \qquad (4)$$

2.1.1 Another Definition of nearly Kahler manifold

If we have a constant α *on* $\overline{M}$ in this manner

$$\|(\overline{\nabla}_X J)(Y)\|^2 = \alpha\left[\|X\|^2\|Y\|^2 - G(X,Y)^2 - G(X,JY)^2\right] \qquad (5)$$

then above type of manifold is recognized as a neatly Kahler manifold of constant type α.

2.2 The Relation between normal connection and the normal bundle of M

If we denote *M* as an m-dimensional Riemannian manifold isometric immersed in a nearly Kahler manifold $\overline{M}$. We define the induced metric on *M* by the same latter G. $\nabla^{\perp}$ is a normal connection which we obtain by inducing a Riemannian connection on *M*. Let be the normal boundle of *M*. Then the relation between normal connection and the normal boundle of *M* is given by

$$\overline{\nabla}_X Y = \nabla_X Y + H(X,Y) \qquad (6)$$

$$\overline{\nabla}_X N = -A_N X + \nabla_X^{\perp} N, X, Y \in \chi(M), N \in \nu \qquad (7)$$

Where, *H* and A_N are recognized as the second fundamental forms.

The relation between these forms are established by the equation

$$G(H(X,Y),N) = G(A_N X,Y) \qquad (8)$$

2.3Another important result of CR-Sub manifoId

The equations of Gauss Codazzi and Ricci are given by

$$R(X,Y;Z,W) = R(X,Y;Z,W) + G(H(X,Z),H(Y,W)) - G(H(Y,Z),H(X,W)) \qquad (9)$$

$$[R(X,Y)Z]^{\perp} = (\overline{\nabla}_X H)(Y,Z) - (\overline{\nabla}_Y H)(X,Z) \qquad (10)$$

$$R(X,Y;N,N^{\perp}) = R^{\perp}(X,Y;N,N^{\perp}) - G([A_N, A_N^{\perp}](X),Y) \qquad (11)$$

If there exists a pair of orthogonal complementary distributions $(D, D^{\perp})$ on *M* satisfying the condition that *D* be holomorphic i.e. *JD=D*, then *M* is said to be a CR-submanifold of $\overline{M}$.

We also observe that dim *D* is even i.e. 2*p* and let $D^{\perp} = q$

We give notation of projection operators *by p* and *q* on *D* and $D^{\perp}$ respectively. The distribution *D* is referred as horizontal distribution and $D^{\perp}$ is called the vertical distribution.

Thus, for $X \in \chi(M)$, we have *X+pX=qX*. We have a another result for a normal vector field N.

$$JN = BN + CN \qquad (12)$$

Where, BN and CN are vertical and normal parts of *JN* respectively.

3. The Important Basic Deductions

In present section, we facilitate some important basic deductions as follows:

3.1.1 Lemma 3.1

We have to prove that if $\overline{M}$ be a 2n-dimensional nearly Kahler manifold of constant holomorphic sectional curvature C, then its Ricci tensor is given by

$$\overline{R}ic(X,Y) = \frac{3}{4}(n+1)CG(X,Y) \qquad (13)$$

3.1.2 Proof

We know that a nearly Kahler manifold $\overline{M}$ of constant holomorphic sectional curvature is an Einstien space. Thus,

$$\overline{R}ic(X,Y) = KG(X,Y) \qquad (14)$$

Where, K is a constant. From equation (2.3), we have

$$\overline{R}ic(X,Y) = \left(\frac{n+1}{2}\right)CG(X,Y) + \frac{3}{2}\sum_{i=1}^{n} G[(\overline{\nabla}_X J)(e_i)(\overline{\nabla}_Y J)(G)] \qquad (15)$$

Finally, from equations (3.2) and (3.3) we get

$$\sum_{i=1}^{n} G[(\overline{\nabla}_X J)(e_i)(\overline{\nabla}_Y J)(e_i)] = \left[\frac{2}{3}K - \left(\frac{n+1}{3}\right)C\right]G(X,Y) \qquad (16)$$

Let us consider the tensor R* which is defined by Takamatsu (1971).

$$R^*(X,Y) = \sum_{i=1}^{n} \overline{R}(X, JY; Je_i, e_i)$$

We also know that

$$g(\overline{R}ic - R^*, \overline{R}ic - 5R^*) = 0 \qquad (17)$$

Thus, from equations (2), (14), (16) and the definition of R*, we find

$$R^*(X,Y) = \left[-\frac{1}{3}K + \frac{2}{3}(n+1)C\right]G(X,Y) \qquad (18)$$

Using (14) and (18) in (16) we obtain

$K = \frac{3}{4}(n+1)C$ Which, proves the lemma

From (16), we also obtain

$$\sum_{i=1}^{n} \|(\overline{\nabla}_X J)(e_i)\|^2 = \frac{1}{2}(n+1)C\|X\|^2 \qquad (19)$$

3.2.1 Lemma

If *M* be a CR - submanifolds of a nearly Kahler manifold $\overline{M}$. Then some results are deducted which are given as follows

$$p(\nabla_X JpY) + p(\nabla_Y JpX) - p(A_{JqY} X) - p(A_{JqX} Y) = Jp_{\nabla_X} Y + Jp_{\nabla_Y} X \qquad (20)$$

$$q(\nabla_X JpY) + q(\nabla_Y JpX) - q(A_{JqY} X) - Q(A_{JqX} Y) = 2BH(X,Y) \qquad (21)$$

$$H(X,JpY) + H(Y,JpX) + \nabla_x^{\alpha} JqY + \nabla_x^{\alpha} JqX = Jq\nabla_x Y + Jq\nabla_y X + 2CH(X,Y) \qquad (22)$$

<u>**3.2.2 Proof**</u>

This is a direct result from the definition of nearly Kahler manifold by equating horizontal, vertical and normal components.

3.3 Lemma

If $\overline{M}$ be a nearly Kahler manifold then we get the standard result given as follows :

$$(\overline{\nabla}_X J)(Y) + (\overline{\nabla}_{JX} J)(JY) = 0 \qquad (23)$$

$$N(X,Y) = -4J[(\overline{\nabla}_X J)(Y)], X, Y \in \chi(\overline{M}) \qquad (24)$$

Where *N(X, Y)* denotes the Neijenhuis tensor.

3.4.1 Lemma

Let *M* denote a CR-submanifold of a nearly Kahler manifold $\overline{M}$ and the condition is that the distribution *D* must be involutive, then we get following results :

$$H(X,JY) = H(JX,Y); X,Y \in D \qquad (25)$$

3.4.2 Proof

For this purpose, we use equations (24) and (6) and finally we obtain

$$\nabla_x JY - \nabla_y JX + H(X,JY) - H(JX,Y) = \frac{1}{2}JN(X,Y) + J[X,Y]$$

We know that by the definition of *N(X,Y),D* is involutive. By using this property, we observe that $JN(X,Y) \in D$ and $J[X,Y] \in D$.

3.5 Lemma

If we have $A_N X \in D$ for each $X \in D$ and $N \in \nu$ the a CR-submanifold *M* of a nearly Kahler manifold $\overline{M}$ is mixed totally geodesic.

3.5.1 Terminology used

3.5.1.1Foliate CR-submanifold

If a CR-submanifold of a nearly Kahler manifold $\overline{M}$ have involutive horizontal distribution D, then such type of manifold is called foliate CR-submanifold.

3.5.1.2 Mixed Totally Geodesic Manifold

If the condition $H(X,Y)=0, X \in D, Y \in D^{\perp}$ fulfills then such type of CR-submanifold is recognized as mixed totally geodesic.

Let *M* be a mixed totally geodesic foliate CR-submanifold of a nearly Kahler manifold $\overline{M}$. Then

$A_N JX = -JA_N X,\ X \in D$ and $N \in \nu$ (26)

3.5.2 Proof

We have $A_N X \in D, JA_N X \in D$ and by using (25), we get

$$G(JA_N X,Y) = -G(A_N X, JY) = -G[H(X,JY),N] = -G[H(JX,Y),N] = -G[A_N JX, Y]$$

Thus, by using lemma (16) and above written equation we have

$H(JX,Y) = CH(X,Y)$

Implementing $JH(X,Y)=BH(X,Y)+CH(X,Y)$ in above-mentioned equation, we obtain

$H(JX,Y) = JH(X,Y) - BH(X,Y)$

If we use the concept of parallelism of horizontal distribution D in equation (21), we get

$BH(X,Y)=0$, This proves the fact (26).

If we use equation (26) conversely in equation (22), we obtain

$$\nabla_X Y + \nabla_Y X \in D$$

Thus, we prove converse that *D* must be parallel.

3.6.1 Lemma

The equation

$H(X,JY) = H(JX,Y) = JH(X,Y); X,Y \in D D$ (27)

holds if *M* be a CR - submanifold of a nearly Kahler manifold $\overline{M}$ with parallel horizontal distribution D.

3.6.2 Lemma

The converse of its also holds if parallel horizontal distribution D is involutive.

3.6.2.1 Terminology used in Lemma

3.6.2.2 Parallel Horizontal Distribution

If we have $\nabla_X Y \in D$ for $X, Y \in D$, then the horizontal distribution *D* is referred as parallel horizontal distribution.

3.6.3 Proof of Lemma

We know that distribution *D* is parallel, then it will be involutive.

Thus, by using deduction (3.4) and equation (27) we have

$H(JX,Y)=CH(X,Y)$

Implementing $JH(X,Y) = BH(X,Y) + CH(X,Y)$ in above mentioned equation, we obtain

$H(JX,Y)=JH(X,Y) - BH(X,Y)$

If we use the concept of parallelism of horizontal distribution *D* in equation (3.9), we get

$BH(X,Y) = 0$. This proves fact (27).

If we use equation (27) conversely in equation (22), we obtain

$$\nabla_X Y + \nabla_Y X \in D$$

Thus, we prove converse that *D* must be parallel.

3.7.1 Deduction

The equation

$A_{JN} X = JA_N X,\ X \in D, N \in \mu$ (28)

Holds, if *M* be a mixed totally geodesic CR-submanifold of a nearly Kahlerian manifold $\overline{M}$ with parallel horizontal distribution D.

3.7.2 Proof of Deduction

By using equation (3.15)

$$G(JA_N X, Y) = -G(A_N X, JY) = -G[H(X,JY),N]$$
$$= -G[JH(X,Y),N] = [G\{H(X,Y)\}JN] = G(A_{JN} X, Y)$$

which proves equation (28).

4. A study of CR-submanifolds of Kahlerian product Manifold

In this section, we analyze the properties of CR-submanifolds of Kahlerian product manifold $\overline{M}$. We also discuss particular cases of Kahlerian product manifold.

4.1Notations and Terminology

The underlying notations related with our study are outlined as follows: $\overline{M}^p =$ A complex p-dimensional Kahlerian manifold.

$\overline{M}^q =$ A complex q-dimensional Kahlerian manifold.

$J_p =$ Almost complex structures of $\overline{M}^p$

$J_q =$ Almost complex structures of $\overline{M}^q$

s_1 & s_2 = Constant holomorphic sectional curvatures.

T_1 = Projection operator of the tangent space of $\overline{M}$ to the tangent space of $\overline{M}^p$.

T_2 = Projection operator of the tangent space of $\overline{M}$ to the tangent space of $\overline{M}^q$.

T= Almost product structure on $\overline{M}$

T_xM=The tangent space of CR-submanifold of Kahlerian manifold $\overline{M}$ at $x \in M$

The following terminology is employed to discuss properties of CR-submanifolds *M* of Kahlerian product manifold $\overline{M}$

If there holds a differentiable distribution

$D : x - D_x \subset T_x M$; where $T_x M$ refers the tangent space of CR-submanifold M of $\overline{M}$ at $x \in M$ concerning with the following conditions :

(a) Distribution D_X must be homomorphic
i.e. $JD_x = D_x$ for each $x \in M$

(b) The complementary orthogonal distribution
$D_x^{\perp} : x - D_x^{\perp} \subset T_x M$ is totally real i.e. $JD_x^{\perp} \subset T_x^{\perp} M$ for each $x \in M$

then, a submanifold having above-mentioned criteria is referred as a CR- submanifold of a Kahlerian product manifold $\overline{M}$.

4.2Analysis of CR-submanifold of Kahlerian Product Manifold

Let us consider that $\overline{M}^p$ and $\overline{M}^q$ be the complex space forms with constant holomorphic sectional curvatures s_1 and s_2. We denote them by notations $\overline{M}^p(s_1)$ and $\overline{M}^q(s_2)$ respectively.

Riemannian curvature tensor $\overline{R}_p$ of $\overline{M}^p(s_1)$ is given by

$$\bar{R}_p(X,Y)Z=\frac{1}{4}s_1[G_p(Y,Z)X-G_p(X,Z)Y+G_p(J_pY,Z)J_pX$$
$$-G_p(J_pX,Z)J_pY+2G_p(X,J_pY)J_pZ \qquad (29)$$

Similiarly, we may write Riemannian curvature tensor $\bar{R}_q$ of $\bar{M}^q(s_2)$ as follows

$$\bar{R}_q(X,Y)Z=\frac{1}{4}s_2[G_q(Y,Z)X-G_q(X,Z)Y+G_q(J_qY,Z)J_qX$$
$$-G_q(J_qX,Z)J_qY+2G_q(X,J_qY)J_qZ \qquad (30)$$

Let us assume that the Kahlerian product manifold be $\bar{M}=\bar{M}^p(s_1)\times\bar{M}^q(s_2)$.
again, we have
$T_1^2=T_1=T_2;T_1T_2=T_2T_1=0$
let us assume $T=T_1T_2$
and it may be verified that $T^2=I$
Thus, finally T is almost product structure on $\bar{M}$.
Further, we give a Riemannian metric G on $\bar{M}$ by
$G(X,Y) = G_p(T_1X,T_1Y) + G_q(T_2X,T_2Y) \qquad (31)$
For any vector field X and Y of $\bar{M}$, it also proves that
$G(TX,Y)=G(TY,X)$
We substitute $JX=J_pT_1X+J_qT_2X$ for any vector field X of $\bar{M}$. Thus, we observe that $J_pT_1=T_1J;\ J_qT_2=T_2J;\ TJ=JT$
$J^2=-1;\ \ G(JX,JY)=G(X,Y);\nabla_XJ=0$
Finally, we conclude that J is a Kahlerian structure on $\bar{M}$.
Let us denote that G *be* the metric tensor field on $\bar{M}$ as well as that induced on M.
$\bar{\nabla}(resp\nabla)$ denotes the covariant differentiation with respect to the Levi-Civita connection.
Then, we calculate the Gauss and Weingarten formulas for M as given below:

$$\bar{\nabla}_XY=\nabla_XY+H(X,Y);\bar{\nabla}_XN=-A_NX+\nabla\frac{1}{X}N \qquad (32)$$

Let H be the second fundamental form (resp. tensor) of M in $\bar{M}$ and $\nabla^{\perp}$ denotes the operator of the normal connection.
So we have,

$$G[H(X,Y),N]=G[A_NX,Y] \qquad (33)$$

4.3 Some Performance Indices of a Kahlerian Product Manifold

In present section, we evaluate some performance indices of Kahlerian product Manifold under the light of above-mentioned analysis.

4.3.1 Riemannian Curvature Tensor

We obtain the Riemannian curvature tensor $\bar{R}$ of a Kahlerian product manifold $\bar{M}$ as follows :

$$\bar{R}(X,Y,Z,W)=\frac{1}{16}(s_1+s_2)[G(Y,Z)G(X,W)-G(X,Z)G(Y,W)$$
$$+G(YJ,Z)G(JX,W)-G(JX)G(JY,W)$$
$$+2G(X,JY)G(JZ,W)+2G(TY,Z)G(TX,W)$$
$$-G(TX,Z)G(TY,W)+G(TJY,Z)G(TJX,W)$$
$$-G(TJY,Z)G(TJY,W)+2G(TX,JY)G(TJZ,W)]$$
$$+\frac{1}{16}(s_1-s_1)[G(TY,Z)G(TY,Z)G(X,W)-G(TX,Z)G(Y,W)]$$
$$+G(Y,Z)G(TX,W)-G(X,Z)G(TY,W)$$
$$+G(TJY,Z)G(JX,W)-G(TJX,Z)G(ZY,W)$$
$$+G(JY,Z)G(TJX,W)-G(JX,Z)G(FJY,W)$$
$$+2G(TX,JY)G(JZ,W)+2G(T,JY)G(TJZ,W)] \qquad (34)$$

For $X,Y,Z,W\in T\bar{M}$

4.3.2 Gauss's Equation

The equation of Gauss is given by

$$R(X,Y,Z,W)=\bar{R}(X,Y,Z,W)+G[H(X,W),H(Y,Z)-G\{H(X,Z)H(Y,W)\}] \qquad (35)$$

Where $R(resp.\bar{R})$ is curvature of $M(resp.\bar{M})$.

4.3.3 The curvature Tensor of M

Let us assume that M be a CR-submanifold of a Kahlerian Product Manifold $\bar{M}$. Thus by implementing Gauss's equation (4.7), the curvature tensor of M is given by

$$R(X,Y,Z,W)=\frac{1}{16}(s_1+s_2)[G(Y,Z)G(X,W)-G(X,Z)G(Y,W)$$
$$+G(JT_1Y,Z)G(JT_1X,W)-G(JT_1X,Z)G(JT_1Y,W)$$
$$+2G(X,JT_1Y)G(JT_1Z,W)+2G(TY,Z)G(TX,W)$$
$$-G(TX,Z)G(TY,W)+G(TJT_1Y,Z)G(TJT_1X,W)$$
$$-G(TJT,X,Z)G(TJT,Y,W)+2G(TX,JT_1Y)G(TJT_1Z,W)]$$
$$+\frac{1}{16}(s_1-s_2)[G(TY,Z)G(X,W)-G(TX,Z)G(Y,W)$$
$$+G(Y,Z)G(TX,W)-G(X,Z)G(TY,W)$$
$$+G(TJT_1Y,Z)G(JT_1X,W)-G(TJT_1X,Z)G(JT_1Y,W)$$
$$+G(JT_1Y,Z)G(TJT_1X,W)-G(JT_1X,Z)G(TJT_1Y,W)$$
$$+2G(TX,JT_1Y)G(JT_1Y,W)+2G(T,JT_1Y)G(TJT_1Z,W)$$
$$+G\{H(X,W),H(X,Z)\}-G\{H(X,Z),H(Y,W)\}]$$

For $X,Y,Z,W\in TM$

5. Some Particular Cases

In this section, we give a brief definition of submanifolds which we obtain under some defined restrictions.
We employ some restrictions on differentiable Distribution *D*.

5.1 Case 1

If $\dim.D_X^{\perp}=0(resp.D_X=0)$ then the CR-submanifold M is referred as holomorphic submanifold or resp. totally real submanifold of $\bar{M}$.

5.2 Case 2

If we treat differentiable Distribution D as orthogonal distribution, then the submanifold Mis behaved as endowed with two complementary orthogonal distribution D and $D^{\perp}$ of real dimensions.

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